3.3 Problems Part 2

Give Examples of...

Question 1. If possible, give an example of a function f that satisfies:

- f is decreasing on $[1, 5]$
- f is concave up on $[1, 5]$.

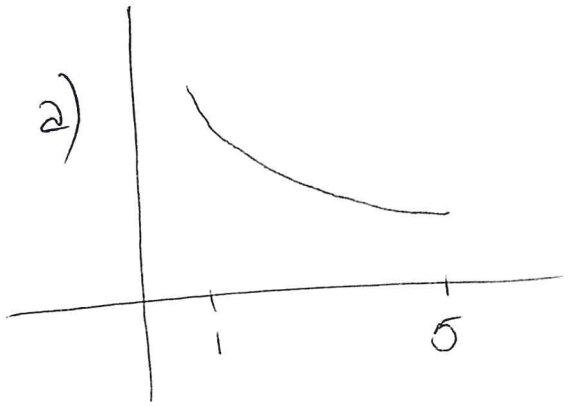
Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.

a)



b) $f = \frac{1}{x}$.

Question 2. If possible, give an example of a function f that satisfies:

- f is increasing on $[1, 5]$
- f is concave up on $[1, 3)$ and is concave down on $(3, 5]$.

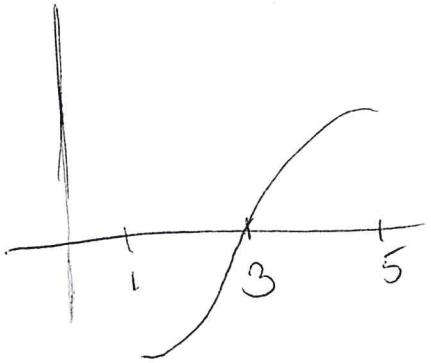
Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.

a)



b) $f(x) = -(x-3)^3 + 12(x-3)$.

Question 3. If possible, give an example of a function f that satisfies:

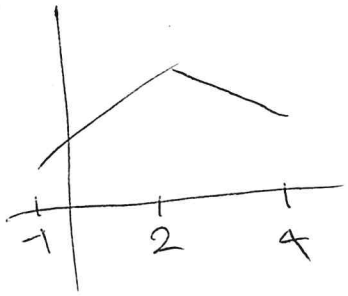
- f is increasing on $[-1, 2)$ and decreasing on $(2, 4]$.
- f is neither concave up or concave down on $[-1, 4]$.

Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.



$$f(x) = \begin{cases} x & x \in [-1, 2) \\ 4 - x & x \in [2, 4] \end{cases}.$$

Question 4. If possible, give an example of a function f that satisfies:

- f is neither increasing nor decreasing on $[1, 4]$.
- f is concave up on $[1, 4]$.

Represent f as

(a) A sketch of a graph

(b) An equation

If it is not possible explain why.

If it is ~~increasing~~ concave up on some portion of $[1, 4]$ it must increase or decrease on this interval.

Question 5. Consider the function $f(x) = \frac{1}{x} + \frac{x}{16}$. Find where:

(a) f is increasing/decreasing

(b) f is concave up/down

(c) Identify any local mins, local maxes, and inflection points.

$$f' = \frac{1}{-x^2} + \frac{1}{16}$$

$$f'' = \frac{2}{+x^3}$$

$f' = 0 \Rightarrow \frac{1}{x^2} = \frac{1}{16} \Rightarrow x = \pm 4.$

$f'' > 0$ for $x > 0$; $f'' < 0$ for $x < 0$.

$f' > 0$ on $(-\infty, -4) \cup (4, \infty)$

$f' < 0$ on $(-4, 0) \cup (0, 4)$.

(a) f is increasing on $(-\infty, -4) \cup (4, \infty)$.
f is decreasing on $(-4, 0) \cup (0, 4)$.

(b) f concave up on $(0, \infty)$
f concave down on $(-\infty, 0)$

(c) local max: $x = -4$ local min: $x = 4$
inflection point $x = 0$

Question 6. Given

$$f'(x) = \frac{(x-6)(x-1)}{(x+3)}, \qquad f''(x) = \frac{(x+9)(x-3)}{(x+3)^3}, \qquad f(-3) \text{ not defined.}$$

determine the intervals on which $f(x)$ increases/decreases, the intervals on which the function is concave up/down and the x values in which the function has maximum, minimum and inflection.

$f' > 0$ on $(-\infty, -3) \cup (6, \infty) \cup (-3, 1)$

$f' < 0$ on $(1, 6)$.

$f'' > 0$ on $(3, \infty) \cup (-9, -3)$

$f'' < 0$ on $(-\infty, -9) \cup (-3, 3)$.

local max @ $x = 1$ local min @ $x = 6$

inflection points @ $x = -9, -3, 3$

Question 7. Determine A and B so that the curve $y = Ax^{1/9} + Bx^{-1/9}$ has an inflection point at $(1, 2)$.

$(x, y) = (1, 2)$

plug into eq

$\hookrightarrow A\,1^{1/9} + B\,1^{-1/9} = 2 \Rightarrow A + B = 2.$

$y' = A\frac{1}{9}x^{-8/9} + B\left(-\frac{1}{9}\right)x^{-10/9}$

$y'' = A\left(-\frac{8}{81}\right)x^{-\frac{17}{9}} + B\left(\frac{10}{81}\right)x^{-\frac{19}{9}} \Rightarrow$ $y''=0$ $x=1$ $\uparrow$

$-8A + 10B = 0$

$\hookrightarrow B = \frac{4}{5}A.$

$\frac{9}{5}A = A + \frac{4}{5}A = 2$

$\hookrightarrow A = \frac{10}{9}.$

$B = \frac{8}{9}$ ⁉