3.4 Problems

Level 1 Problems

Question 1. Evaluate the limit if it exists:

(a) $\lim\limits_{x\to\infty} \dfrac{x+1}{3-2x}$

$$\lim_{x\to\infty} \frac{1+1/x}{3/x-2} = \frac{\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)}{\lim\limits_{x\to\infty}\left(\frac{3}{x}-2\right)} = -\frac{1}{2}.$$

(b) $\lim\limits_{x\to\infty} \dfrac{x^2+x-5}{3x-2\sqrt{x^3}}$

$$\lim_{x\to\infty} \frac{x^{1/2}+x^{-1/2}-5x^{-3/2}}{3x^{-\frac{1}{2}}-2} = \frac{\lim\limits_{x\to\infty}\left(x^{1/2}+x^{-1/2}-5x^{-3/2}\right)}{\lim\limits_{x\to\infty}\left(3x^{-\frac{1}{2}}-2\right)}$$

$$= \frac{\infty}{-2} = -\infty$$

(c) $\lim\limits_{x\to-\infty} \dfrac{x^2+x-5}{3x-2\sqrt{|x|^3}} =$

$$= \frac{\lim\limits_{x\to\infty}\left(\frac{x^2}{|x|^{3/2}}+|x|^{-1/2}-5|x|^{-3/2}\right)}{\lim\limits_{x\to\infty}\left(3|x|^{-1/2}-2\right)}$$

$$= -\infty$$

Quick Conceptual/Fun Questions

Question 2. What is the maximum number of vertical asymptotes that a function can have?

∞ for example, $\tan x$

Question 3. What is the maximum number of horizontal asymptotes that a function can have?

2 * $x \to \infty$ & $x \to -\infty$.

Question 4. Give an example of a function $g(x)$ so that $f(x) = \dfrac{3\sqrt[3]{x^2} + 5\sqrt{x^3} + 7\sqrt[5]{x^9}}{xg(x)}$ has a horizontal asymptote $y = 6$.

we need only consider leading term of the numerator $7x^{9/5}$

let $g(x) = \frac{7}{6} x^{4/5}$

Then.

$\lim_{x \to \infty} f(x) = 6$.

Level 2+ Problems

Question 5. Find the horizontal asymptote(s) of the following functions if they exist.

(a) $f(x) = \dfrac{(x+1)(2x-5)}{(3x-1)(1-x)}$

leading term of numerator : $2x^2$

" " " denominator: $-3x^2$.

horizontal asymptote @ $x = \pm\infty$ is $y = -\frac{2}{3}$.

(b) $g(x) = \dfrac{(x+1)(2x-5)}{|3x^2|}$

leading term of numerator $2x^2$

" " " denominator $3x^2$.

horizontal asymptote @ $x = \pm\infty$ is $y = \frac{2}{3}$.

(c) $h(x) = \dfrac{\sqrt{3x^6+5x+1}}{(x^2+1)(2x-5)} \dfrac{\frac{1}{x^3}}{\frac{1}{x^3}} = \dfrac{\sqrt{3+5x^{-5}+x^{-6}}}{(1+\frac{1}{x^2})(2-\frac{5}{x})}$

$$\lim_{x\to\infty} h(x) = \lim_{x\to-\infty} h(x) = \sqrt{3}.$$

Question 6. Find the horizontal asymptote(s) of the following functions if they exist.

(a) $f(x) = \sqrt{9x^2 + x} - 3x$

$$\left(\sqrt{9x^2+x} - 3x\right)\left(\frac{\sqrt{9x^2+x} + 3x}{\sqrt{9x^2+x} + 3x}\right) = \frac{9x^2 + x - 9x^2}{\sqrt{9x^2+x} + 3x}$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x}{\sqrt{9x^2+x} + 3x} = \lim_{x\to\infty} \frac{1}{\sqrt{9+\frac{1}{x}} + 3} = \frac{1}{6}.$$

(b) $g(x) = \sqrt{x^2 + x + 1} + x$

$$\lim_{x\to\infty} g(x) = \infty$$

(c) $h(x) = 3x \sin \dfrac{2}{5x}$

$$h(x) = 3x \sin \frac{2}{5x} = f(y) = \frac{3}{y} \sin\left(\frac{2}{5}y\right) \text{ for } y = \frac{1}{x}.$$

$$\lim_{x\to-\infty} h(x) = \lim_{x\to\infty} h(x) = \lim_{y\to 0} f(y) = \frac{3\cdot 5}{2} \lim_{y\to 0} \frac{\sin\frac{2}{5}y}{\frac{2}{5}y} = 1.$$