3.5a Problems

Example 1. How does a slant asymptote differ from a verticle asymptote? from a horizontal asymptote?

Vertical asymptote → $|f| \to \infty$ at finite value of x

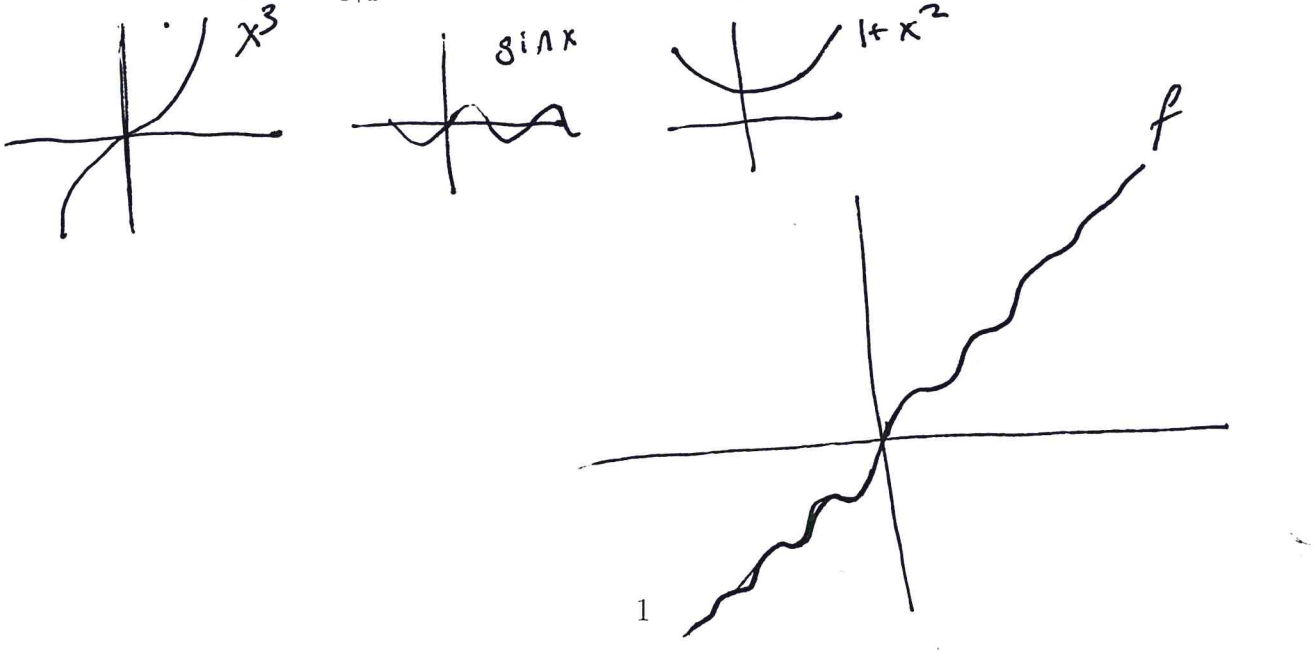
Horizontal asymptote → $f \to$ finite value as $|x| \to \infty$

slant asymptote → $|f| \to \infty$ as $|x| \to \infty$
and the growth is linear.

Example 2. (a) Plot the graph of $f(x) = x + \sin x$, does it have a slant asymptote?

"Linear part of f" is x remainder is $\sin(x)$. But $\lim_{x\to\infty} \sin x = \text{DNE}$,
so there is no slant asymptote.

(b) Plot the graph of $f(x) = \frac{x^3+\sin x}{1+x^2}$, does it have a slant asymptote?



Example 3. Practice long division of the following rational expressions.

(a) $\frac{x^4}{x^2+1}$

$$x^2+1 \overline{)\, x^4}$$
$$x^4 + x^2$$
$$-x^2$$
$$-x^2 - 1$$
$$+1$$

$$\frac{x^4}{x^2+1} = x^2 - 1 + \frac{1}{x^2+1}$$

(b) $\frac{x^5+2x^4+3}{x^2+1}$

$$x^2+1 \overline{)\, x^5+2x^4+3}$$
$$x^5 + x^3$$
$$-x^3+2x^4+3$$
$$2x^2 + 2x^4$$
$$-x^3-2x^2+3$$
$$-x^3 - x$$
$$-2x^2+x+3$$
$$-2x^2 - 2$$
$$-x+5$$

$$\frac{x^5+2x^4+3}{x^2+1} = x^3+2x^2-x-2 + \frac{x+5}{x^2+1}$$

$\frac{x^3+3x^2+5}{x^2+1}$

$$x^2+1 \overline{)\, x^3+3x^2+5}$$
$$x^3 + x$$
$$3x^2-x+5$$
$$3x^2 + 3$$
$$-x+2$$

$$\frac{x^3+3x^2+5}{x^2+1} = x+3 + \frac{-x+2}{x^2+1}$$

Example 4. Find the slant asymptote of the function $f(x) = \dfrac{x^3+4x^2+x-8}{x^2-x-2}$

$$x^2-x-2 \overline{)\, x^3+4x^2+x-8}$$
$$x^3 - x^2 - 2x$$
$$5x^2+3x-8$$
$$5x^2 - 5x - 10$$
$$8x+2$$

$$\frac{x^3+4x^2+x-8}{x^2-8-2} = x+5 + \frac{8x+2}{x^2-x-2}.$$

∴ slant asymptote:

$$y = x+5.$$