3.5 Problems

Curve Sketching

Example 1. How can you tell if a function is even or odd

(a) by looking at the equation?

(i) $f(x)$ is even if $f(x) = f(-x)$

(ii) $f(x)$ is odd if $f(x) = -f(-x)$

(iii) If (i) or (ii) do not hold f is neither even or odd.

(b) by looking at the graph?

(i) f is even if it is symmetric around line $x = 0$

(ii) f is odd if it has 180° rotation symmetry.

its graph

Example 2. How can you tell the difference between a vertical asymptote and a hole in a graph by looking at the equation of a function?

For rational functions, a hole arises if both numerator + denominator have zero at same x. Otherwise a zero in the denominator implies a vertical asymptote.

Example 3. Use "the guidelines" in your notes to sketch a detailed graph of each of the following:

(a) $f(x) = x^3 - 12x^2 + 36x$

$f' = 3x^2 - 24x + 36$. Critical points $f' = 0$

$f'' = 6x - 24$. $\hookrightarrow x^2 - 8x + 12 = 0$

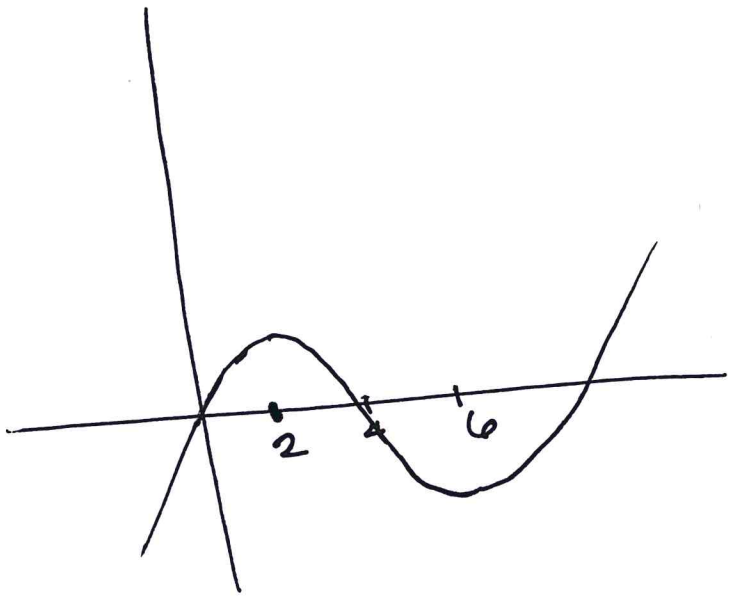
$x = \frac{8 \pm \sqrt{64 - 48}}{2} = 6, 2$

Inflection point $f'' = 0 \leftrightarrow x = 4$.

$f'' > 0$ on $(4, \infty)$

$f'' < 0$ on $(-\infty, 4)$

local max $(2, f(2))$ local min $(6, f(6))$



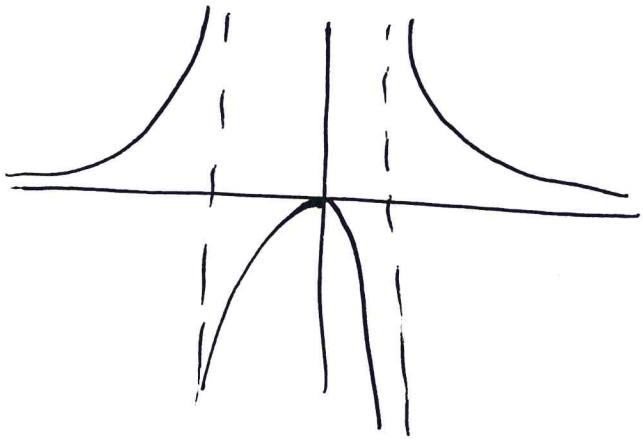
(b) $f(x) = \dfrac{2x^2}{x^2-1}$

$$f' = \frac{4x(x^2-1) - 2x^2(2x)}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2}$$

$$f' = 0 \iff x = 0 \qquad f' \begin{cases} < 0 & x > 0 \\ > 0 & x < 0 \end{cases}$$

$$f'' = \frac{-4(x^2-1)^2 + 4x\,(2 \cdot 2x\,(x^2-1))}{(x^2-1)^4} = \frac{4(3x^2+1)(x^2-1)}{(x^2-1)^4}$$

$$f'' \begin{cases} < 0 & \text{on } x \in (-1,1) \\ > 0 & \text{on } x \in (-\infty,-1) \cup (1,\infty) \end{cases}$$



local max @ $(0, f(0))$