## 4.4 Problems

**Question 1.** Evaluate the general indefinite integral.

(a) $\int (u+2)(3-u)\, du$ $= \int (u3 + 6 - u^2 - 2u)\, du$

$= \int (6 + u - u^2)\, du = 6u + \frac{1}{2}u^2 - \frac{1}{3}u^3 + C$

(b) $\int \frac{\sin 2x}{\cos x} dx$ Recall: $2\cos x \sin x = \sin 2x$

$\int \frac{\sin 2x}{\cos x} dx = \int \frac{2\cos x \sin x}{\cos x} dx = \int 2\sin x\, dx = -2\cos x + C$

**Question 2.** Verify that the formula: $\int \cos^3 x\, dx = \sin x - \frac{1}{3}\sin^3 x + C$ is correct.

Take derivative of bothsides

$\frac{d}{dx}\left(\sin x - \frac{1}{3}\sin^3 x + C\right) = \cos x - \sin^2 x \cos x = \cos x(1 - \sin^2 x)$

$= \cos^3 x$

$\frac{d}{dx} \int \cos^3 x\, dx = \cos^3 x.$

**Question 3.** If $h'(t)$ the the rate of growth of Ryan in inches/year what does $\int_5^8 h'(t)\,dt$ represent?

$\int_5^8 h'(t) \equiv$ Total change of height from age 5 to 8.

**Question 4.** Water flows from the bottom of a storage tank at a rate of $r(t) = 10 - 2t$ liters per minute, where $0 \le t \le 5$.

(a) After a minute a 10 liter bucket is placed under the storage tank to catch the water. How long until the bucket starts to overflow?

Let $V(t)$ be amount of water in Bucket.

Then $V'(t) = 10 - 2t$ and $V(0) = 0$.

$\Rightarrow V = 10t - t^2 + C$ ; $V(0) = C = 0$ ; $V(t) = 10$ + $t \le 5$

$\Rightarrow 10t - t^2 = 10 \Rightarrow t = \frac{10 \pm \sqrt{10^2 - 40}}{2} \rightarrow t = 5 \pm \sqrt{15} \Rightarrow t = 5 - \sqrt{15}$

(b) At $t = 4$ another 10 liter bucket is placed under the storage tank to catch the water. How much water does this bucket have in it at the end?

$$W = \int_4^5 r(t)\,dt = \int_4^5 (10 - 2t)\,dt$$

$$= (10t - t^2)\Big|_4^5 = (50 - 25) - (40 - 16) = 1.$$