Math 132 Fall 2017
Group Quiz 7 10/27/17 Name (Print): Solutions

1. Consider $f(x) = x^3 - x$. Find all c satisfying the Mean Value Theorem on the interval $[-1, 2]$.

Find $c \in (-1,2)$ so that $f'(c) = \frac{f(2) - f(-1)}{(2)-(-1)} = \frac{(8-2)-(-1+1)}{2+1}$

ie $f'(c) = \frac{6}{3} = 2$.

Solve:

$f'(x) = 3x^2 - 1 = 2$

$\therefore x^2 = 1$

$x = \pm 1$

But $-1 \notin (-1,2)$

$\therefore$ $c = 1$ is the only value of x satisfying MVT.

2. Show $f(x) = 4x + \sin(x)$ has at most one root.

Suppose f has 2 roots. Then ther is $a, b \in \mathbb{R}$, $a \neq b$ so that $f(a) = f(b) = 0$. Then, by MVT there is $c \in (a,b)$ so that $f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0}{b-a} = 0$

But $f'(x) = 4 + \cos x$ so $f'(x) > 0$ for all $x \in \mathbb{R}$

$\therefore$ there is no c so that $f'(c) = 0$. Thus f has at most 1 root.

3. Let $f(x) = x^3 + x^{-2}$ find the local maxima and minima and inflection points, if any.

$f' = 3x^2 - 2x^{-3}$

$0 = f' \Rightarrow 3x^2 = 2x^{-3}$

$x^5 = \frac{2}{3}$

$x = \sqrt[5]{2/3}$

critical points – $x = 0$ & $x = \sqrt[5]{2/3}$.

$f'' = 6x + 6x^{-4}$

$0 = f'' \Rightarrow x^5 = -1$

$\Rightarrow x = -1$

$f'' > 0$ on $(-1,0) \cup (0,\infty)$

$f'' < 0$ on $(-\infty, -1)$

Local min at $x = \left(\frac{2}{3}\right)^{1/5}$

No local max

Inflection point @ $x = -1$.

(Not at $x = 0$)