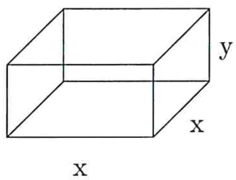
Math 132 Fall 2017
Group Quiz 9 11/08/17 Name (Print): Solutions

1. Consider a cube with a square base and no front and top sides. The total square area of material used is 10 cm^2. What is the maximal volume of the box?



$$\underbrace{x^2}_{\text{base}} + \underbrace{3xy}_{\text{3 sides}} = 10\ cm^2$$

$$y = \frac{10-x^2}{3x}$$

Maximize

$$V = x^2 y = x\,\frac{10-x^2}{3} =$$

$$V' = \frac{10-x^2}{3} + x\,\frac{-2x}{3} = \frac{10-3x^2}{3}$$

$$V' = 0 = \frac{10-3x^2}{3} \longleftrightarrow x_0 = \sqrt{\frac{10}{3}}$$

$$V'' = -6x \quad \therefore \sqrt{\tfrac{10}{3}} \text{ is local max.}$$

$$V(x_0) = \sqrt{\frac{10}{3}}\left(\frac{10-\frac{10}{3}}{3}\right)$$

$$= \sqrt{\frac{10}{3}}\ \frac{20}{9}$$

2. Use Newton's method to approximate $x = \sqrt{8}$. That is, approximate x so that $x^2 - 8 = 0$ using Newton's method. Let $x_0 = 3$ and find the first two iterations x_1, x_2.

$$f(x) = x^2 - 8 ; \quad \text{Newton: } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$f'(x) = 2x \qquad \therefore x_{n+1} = x_n - \frac{x_n^2 - 8}{2x_n}$$

$$= x_n - \tfrac{1}{2}x_n + \frac{4}{x_n}$$

$$= \tfrac{1}{2}x_n + \frac{4}{x_n}.$$

$$x_0 = 3;$$

$$x_1 = \tfrac{1}{2}3 + \frac{4}{3} = \frac{17}{6}$$

$$x_2 = \tfrac{1}{2}\left(\frac{17}{6}\right) + 4/\tfrac{17}{6} = \frac{17}{12} + \frac{24}{17}.$$