Math 132 Fall 2017
Quiz 7 10/27/17 Name (Print): ____________

1. Consider $f(x) = x^4 - 2x^2 + 4x$ on the interval $[-2, 2]$. Find all c satisfying the Mean Value Theorem.

$$f'(c) = \frac{f(2) - f(-2)}{2 - (-2)} = \frac{16}{4} = 4.$$

$$f' = 4x^3 - 4x + 4 = 4 \leftrightarrow x(x^2 - 1) = 0$$

$$\leftrightarrow x = 0, \pm 1.$$

2. Show $f(x) = x^3 + 3x + \sin x$, has at most 1 root.

Suppose f has 2 roots, ie $a \neq b$ and $f(a) = f(b) = 0$.

Then there is $c \in (a, b)$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = 0$$

$$f' = 3x^2 + 3 + \cos x.$$

But $f' > 0$ for all $x \in \mathbb{R}$.

So there is atmost one root of f.

3. Let $f(x) = x^2 - x^{-3}$ find the local minima and maxima and inflection points.

$f' = 2x + 3x^{-4}$

$f'' = 2 - 12x^{-5}$

$f' = 0 \iff x^5 = -3/2 \iff x = -\left(\frac{3}{2}\right)^{1/5}$

$f'' = 0 \iff x^5 = \frac{1}{6} \iff x = 6^{-1/5}.$

$$f' \begin{cases} > 0 & \text{on } x \in \left(-\left(\frac{3}{2}\right)^{1/5}, 0\right) \cup (0, \infty) \\ < 0 & \text{on } x \in \left(-\infty, -\left(\frac{3}{2}\right)^{1/5}\right) \end{cases}$$

$$f'' \begin{cases} > 0 & \text{on } x \in (-\infty, 0) \cup (6^{-1/5}, \infty) \\ < 0 & \text{on } x \in (0, 6^{-1/5}) \end{cases}$$

local min: $x = -\left(\frac{3}{2}\right)^{1/5}$

No local max.

Inflection point: $x = 6^{-1/5}$