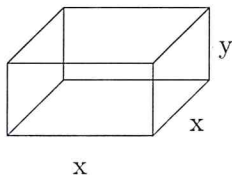


1. Consider a box with a square base and no top side. The total square area of material used is  $20 \text{ cm}^2$ . What is the maximal volume of the box?



$$x^2 + 4xy = 20$$

$$y = \frac{20 - x^2}{4x}$$

Maximize

$$V = x^2 y = x \left( \frac{20 - x^2}{4} \right)$$

$$V' = \frac{20 - x^2}{4} + \frac{-2x^2}{4} = \frac{20 - 3x^2}{4}$$

$$V' = 0 \Leftrightarrow x_0 = \sqrt{\frac{20}{3}}$$

$$V'' = -6x$$

$\therefore x_0$  is local max

$$V(x_0) = \sqrt{\frac{20}{3}} \left( \frac{20 - 3\left(\frac{20}{3}\right)}{4} \right)$$

2. Approximate a root of  $f(x) = x^2 - 3x + 1$  using Newton's method. Let  $x_0 = 0$  and find the first 2 approximations  $x_1, x_2$ .

$$f'(x) = 2x - 3$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 - \frac{1}{-3} = \frac{1}{3}$$

$$x_2 = \frac{1}{3} - \frac{\frac{1}{9} - 1 + 1}{\frac{2}{3} - 3} = \frac{1}{3} + \frac{1}{21} = \frac{10}{21}$$