Orr-Sommerfeld Equation

$$i\alpha RU''\phi + (\zeta R - \lambda^2 - i\alpha RU)(\phi'' - \lambda^2\phi) + \phi'''' - \lambda^2\phi'' = 0 \quad \text{on } [0, \infty)$$

where:

(1) $\phi \in C^4([0, \infty))$ is called an eigenvector and it has to satisfy:
$\phi(0) = \phi'(0) = 0$, $\phi \not\equiv 0$ and
$\phi(x), \phi'(x), \phi''(x), \phi'''(x)$ coverge to 0 as $x \to \infty$

(2) $\zeta \in \mathbb{C}$ is called an **eigenvalue**

(3) $\alpha, \beta \in \mathbb{C}$ are called wave numbers, $\lambda^2 = \alpha^2 + \beta^2$, $i^2 = -1$

(4) $R > 0$ is called the Reynolds number

(5) $U \in C^2([0, \infty))$ is the flow

Theorem (Milan Milavčič) *If for some* $\varepsilon > 0$

$$\sup_{x \geq 0} e^{\varepsilon x} |U'(x)| < \infty$$

then the Orr-Sommerfeld equation has only finitely many eigenvalues. If, in addition,

$$\sup_{x \geq 0} e^{\varepsilon x} |U''(x)| < \infty$$

then there exists $c > 0$*, which depends only on* U*, such that the Orr-Sommerfeld equation has* **no** *eigenvalues if* $|\alpha| R < c$*.*