Supplemental Exercises for Section 2.3

To better understand the formal definition of limit it is essential to use it to prove some obvious limit statements. Prove each of the following limit obvious limit statements using the formal definition of limit.

1. $\lim_{x \to 3} x + 2 = 5$

2. $\lim_{x \to 5} x - 3 = 2$

3. $\lim_{x \to 1} x - 3 = -2$

4. $\lim_{x \to -2} x + 6 = 4$

5. $\lim_{x \to -4} x + 2 = -2$

6. $\lim_{x \to 3} 2x - 4 = 2$

7. $\lim_{x \to 1} 3x - 1 = 2$

8. $\lim_{x \to 2} -3x + 4 = -2$

9. $\lim_{x \to -1} -x - 1 = 0$

The following problems are more difficult.

10. $\lim_{x \to 1} x^2 = 1$

11. $\lim_{x \to -1} x^2 = 1$

12. $\lim_{x \to 2} x^2 - 1 = 3$

Answers to Selected Problems

1. Choose $\delta = \epsilon$.

4. Choose $\delta = \epsilon$.

6. Choose $\delta = \frac{\epsilon}{2}$.

7. Choose $\delta = \frac{\epsilon}{3}$.

10. Choose δ to be the smaller of $1 - \sqrt{1 - \epsilon}$ and $\sqrt{1 + \epsilon} - 1$. Here it's assumed that $0 < \epsilon < 1$.