MIDMICHIGAN OLYMPIAD in MATHEMATICS 2007
7–9 grades

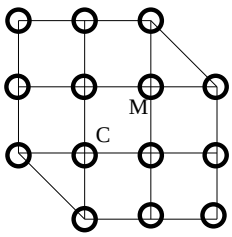
Calculators are not allowed!

1. The Evergreen School booked buses for a field trip. Altogether, 138 people went to West Lake, while 115 people went to East Lake. The buses all had the same number of seats and every bus has more than one seat. All seats were occupied and everybody had a seat. How many seats were on each bus?

2. In New Scotland there are three kinds of coins: 1 cent, 6 cent, and 36 cent coins. Josh has 99 of the 36-cent coins (and no other coins). He is allowed to exchange a 36 cent coin for 6 coins of 6 cents, and to exchange a 6 cent coin for 6 coins of 1 cent. Is it possible that after several exchanges Josh will have 500 coins?

3. Find all solutions a, b, c, d, e, f, g, h if these letters represent *distinct* digits and the following multiplication is correct:

$$\begin{array}{cccccc} & & a & b & c \\ \times & & & d & e \\ \hline & f & a & g & c \\ + & b & b & h & \\ \hline f & f & e & g & c \end{array}$$

4. Is it possible to find a rectangle of perimeter 10 m and cut it in rectangles (as many as you want) so that the sum of the perimeters is 500 m?

5. The picture shows a maze with chambers (shown as circles) and passageways (shown as segments). A cat located in chamber C tries to catch a mouse that was originally in the chamber M. The cat makes the first move, moving from chamber C to one of the neighboring chambers. Then the mouse moves, then the cat, and so forth. At each step, the cat and the mouse can move to any neighboring chamber or not move at all. The cat catches the mouse by moving into the chamber currently occupied by the mouse. Can the cat get the mouse?