

MidMichigan Mathematical Olympiad
April 25, 2009
Solutions Grades 5-6

1. Anne purchased yesterday at WalMart in Puerto Rico 6 identical notebooks, 8 identical pens and 7 identical erasers. Anne remembers that each eraser costs 73 cents. She did not buy anything else. Anne told her mother that she spent 12 dollars and 76 cents at Walmart. Can she be right? Note that in Puerto Rico there is no sales tax.

Solution. Since Anne purchased an even number of notebooks and an even number of pens, she spent an even number of cents on pens and notebooks together. She purchased an odd number of erasers and each eraser costs an odd number of cents. Thus she spent an odd number of cents on erasers. Therefore she spent at Walmart an odd number of cents which means that she could not spend 12 dollars and 76 cents.

2. Two men ski one after the other first in a flat field and then uphill. In the field the men run with the same velocity 12 kilometers/hour. Uphill their velocity drops to 8 kilometers/hour. When both skiers enter the uphill trail segment the distance between them is 300 meters less than the initial distance in the field. What was the initial distance between skiers? (There are 1000 meters in 1 kilometer.)

Solution. When the first skier went uphill but the second skier still ran on a horizontal surface the distance between them shortened with rate 4 kilometers/hour. By the time the second skier has reached the hill the distance has dropped by 300 meters meaning that the time between the first skier has reached the hill and the second skier has reached the hill is $\frac{0.3}{4} = \frac{3}{40}$ hour. The speed of the second skier was 12 kilometers/hour, and so the initial distance between skiers was $\frac{3}{40} \times 12 = 900$ meters.

3. In the equality $** + *** = ****$ all the digits are replaced by *. Restore the equality if it is known that any numbers in the equality does not change if we write all its digits in the opposite order.

Solution. Rewrite $aa+bc9=effe$. Note, that since the result is four-digit number $e=1$. Note also that $b=9$, and $f=0$ since only 1 can be carried over to b to get a four-digit number. We obtain $aa+9c9=1001$. To obtain 1 in the last position of the sum $a=2$, and the answer is $22+979=1001$.

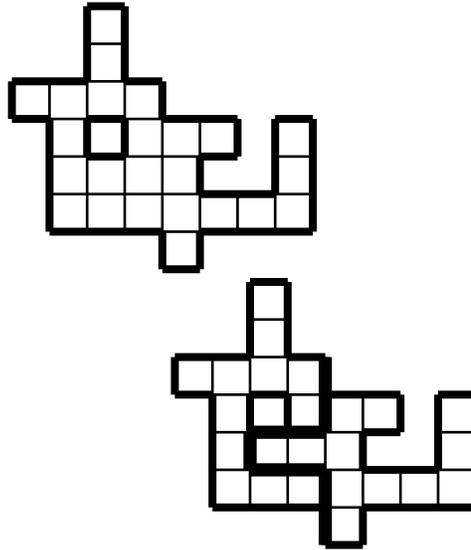
4. If a polyleg has even number of legs he always tells truth. If he has an odd number of legs he always lies. Once a green polyleg told a dark-blue polyleg ”- I have 8 legs. And you have only 6 legs!” The offended dark-blue polyleg replied ”-It is me who has 8 legs, and you have only 7 legs!” A violet polyleg added ”-The dark-blue polyleg indeed has 8 legs. But I have 9 legs!” Then a stripped polyleg started: ”-None of you has 8 legs. Only I have 8 legs!”

Which polyleg has exactly 8 legs?

Solution.Note that if a polyleg claims that it has odd number of legs it lies. So,

- The violet polyleg lies, and the dark-blue does not have 8 legs.
- The dark-blue polyleg lies and it has odd number of legs.
- The green polyleg lies and it can not have 8 legs.
- The stripped polyleg tells the truth and it indeed has 8 legs.

5. Cut the figure shown below in two equal pieces. (Both the area and the form of the pieces must be the same.)



Solution.