

MATH OLIMPIAD 2010

5–6 grades

Solutions

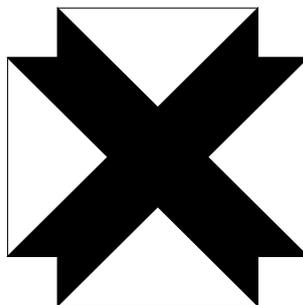
1. Ben and his dog are walking on a path around a lake. The path is a loop 500 meters around. Suddenly the dog runs away with velocity 10 km/hour. Ben runs after it with velocity 8 km/hour. At the moment when the dog is 250 meters ahead of him, Ben turns around and runs at the same speed in the opposite direction until he meets the dog. For how many minutes does Ben run?

Solution: Dog goes away from Ben at $10-8=2$ km/hour. The time to make the distance 250 meters is $(1/4)/2 = 1/8$ hour. After that it takes $1/4/(10 + 8) = 1/72$ hour. Totally it is $1/8 + 1/72 = 5/36$ hour.

2. The six interior angles in two triangles are measured. One triangle is obtuse (i.e. has an angle larger than 90°) and the other is acute (all angles less than 90°). Four angles measure 120° , 80° , 55° , and 10° . What is the measure of the smallest angle of the acute triangle?

Solution: 120° is in the obtuse triangle. Then 80° is not because otherwise the sum of angles is more than 200° . Hence 80° is in the acute triangle. Then 10° is not in the acute, because otherwise the third angle in the acute is a right angle. Then 10° is in the obtuse. And the third angle in the obtuse is 50° . Hence the angles in the acute are 80° , 55° , and 45° .

3. The figure below shows a 10×10 square with small 2×2 squares removed from the corners. What is the area of the shaded region?



Solution: Move the four remaining parts together. They form a 6×6 square. Then the area of the shaded region is $10 \times 10 - 4 \times (2 \times 2) - 6 \times 6 = 100 - 16 - 36 = 48$

4. Two three-digit whole numbers are called relatives if they are not the same, but are written using the same triple of digits. For instance, 244 and 424 are relatives. What is the minimal number of relatives that a three-digit whole number can have if the sum of its digits is 10?

Solution: Note that if the number consists of three distinct digits then it has at least 6 relatives. If the number contains two equal digits and one different digit and none of the digits is zero then it has two relatives. Finally, 505 that has only one relative 550.

5. Three girls, Ann, Kelly, and Kathy came to a birthday party. One of the girls wore a red dress, another wore a blue dress, and the last one wore a white dress. When asked the next day, one girl said that Kelly wore a red dress, another said that Ann did not wear a red dress, the last said that Kathy did not wear a blue dress. One of the girls was truthful, while the other two lied. Which statement was true?

Solution: Assume that the first person tells the truth, then both statements "Kelly wore the red dress" and "Ann did not wear red dress" are true statements violating the condition that only one person tells the truth. Hence the first person is lying. If the second person tells the truth then since the other two persons are lying, Kelly could not have red dress either. Hence Kathy must have red dress which makes the third statement true. We got a contradiction again. The only remaining option is that the third person tells the truth. Then Ann had red dress, Kelly had blue one while Kathy wore white dress. This satisfies all the conditions and we are done.