

SOLUTIONS GRADES 5-6

- The clock is 2 hours 20 minutes ahead of the correct time each week. The clock is set to the correct time at midnight Sunday to Monday. What time does this clock show at 6pm correct time on Thursday?

Answer: 75 minutes. There are $3 \times 24 + 18$ hours from Sunday midnight to Thursday, 6pm. It makes $\frac{3 \cdot 24 + 18}{7 \cdot 24}$ part of the week. Therefore, by the 6pm on Thursday, the clock is $\frac{3 \cdot 24 + 18}{7 \cdot 24} \cdot 140$ minutes ahead of the right time which makes 75 minutes.

- Five cities A, B, C, D , and E are located along the straight road in the alphabetical order. The sum of distances from B to A, C, D and E is 20 miles. The sum of distances from C to the other four cities is 18 miles. Find the distance between B and C .

Answer: The distance is 2 miles. Indeed, the total distance d_B from A, C, D, E to B is $d_B = AB + BC + BD + BE$ while the total distance d_C from A, B, D, E to C is $d_C = (AB + BC) + BC + (BD - BC) + (BE - BC) = AB + BC + BD + BE - BC$. Hence, $BC = d_B - d_C = 2$ [miles].

- Does there exist distinct digits a, b, c , and c such that $\overline{abc} + \bar{c} = \overline{bda}$? Here \overline{abc} means the three digit number with digits a, b , and c . Different letters denote different digits.

Answer: NO. Indeed, since $b \neq a$ it implies that $b = 9$. Moreover, looking at the first position we notice that $b = a + 1$. Therefore, $a = 8$. However, then $c = 9$ which is not possible since different letters mean different digits and 9 is already taken by b , or $c = 4$ which is also impossible because then nothing is carried over to the first and the second place.

- Kuzya, Fyokla, Dunya, and Senya participated in a mathematical competition. Kuzya solved 8 problems, more than anybody else. Senya solved 5 problem, less than anybody else. Each problem was solved by exactly 3 participants. How many problems were there?

Answer: There were 9 problems. Indeed, each of Fyokla and Dunya solved either 6 or 7 problems. Therefore, there are 3 possibilities.

Fyokla solved 6 problems, Dunya solved 6. Totally with Kuzya and Senya we have 25 pairs (participant, problem solved). Note that the number of such pairs must be divisible by 3 since each problem was solved by exactly 3 participants. Therefore this option contradicts initial condition.

If Fyokla solved 6 problems, and Dunya - 7 problems or vice versa, we have 26 pairs which is impossible by similar arguments.

Finally, the only possibility holds when both Fyokla and Dunya solved 7 problems each.

Then we have 27 pairs, or 9 problems.

- Mr Mouse got to the cellar where he noticed three heads of cheese weighing 50 grams, 80 grams, and 120 grams. Mr. Mouse is allowed to cut simultaneously 10 grams from any two of the heads and eat them. He can repeat this procedure as many times as he wants. Can he make the weights of all three pieces equal?

Answer: YES. Mr. Mouse cuts 7 times from 80 gram and 120 gram cheese heads. Then he cuts 4 times from 50 gram and 120 gram heads. After there are three heads left each weighing 10 grams.