MidMichigan Olympiad
2018

Problems
5-6

1. A Slavic dragon has three heads. A knight fights the dragon. If the knight cuts off one dragon's head three new heads immediately grow. Is it possible that the dragon has 2018 heads at some moment of the fight?

2. Peter has two squares 3x3 and 4x4. He must cut one of them or both of them in no more than four parts in total. Is Peter able to assemble a square using all these parts?

3. Usually, dad picks up Constantine after his music lessons and they drive home. However, today the lessons have ended earlier and Constantine started walking home. He met his dad 14 minutes later and they drove home together. They arrived home 6 minutes earlier than usually. Home many minutes earlier than usual have the lessons ended? Please, explain your answer.

4. All positive integers from 1 to 2018 are written on a blackboard. First, Peter erased all numbers divisible by 7. Then, Natalie erased all remaining numbers divisible by 11. How many numbers did Natalie remove? Please, explain your answer.

5. 30 students took part in a mathematical competition consisting of four problems. 25 students solved the first problem, 24 students solved the second problem, 22 students solved the third, and, finally, 21 students solved the fourth. Show that there are at least two students who solved all four problems.

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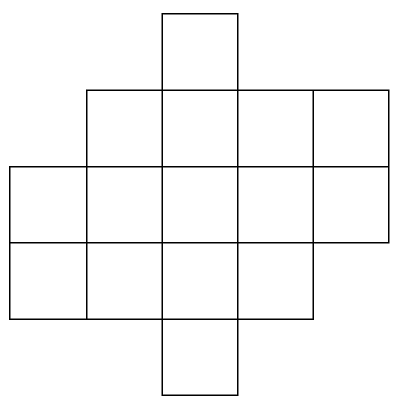
Problems
7-9

1. Is it possible to put 9 numbers 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 in a circle in a way such that the sum of any three circularly consecutive numbers is divisible by 3 and is, moreover:
   a) greater than 9 ? b) greater than 15?

2. You can cut the figure below along the sides of the small squares into



   several (at least two) identical pieces. What is the minimal number of such equal pieces?

3. There are 100 colored marbles in a box. It is known that among any set of ten marbles there are at least two marbles of the same color. Show that the box contains 12 marbles of the same color.

4. Is it possible to color squares of a 8x8 board in white and black color in such a way that every square has exactly one black neighbor square separated by a side?

5. In a basket, there are more than 80 but no more than 200 white, yellow, black, and red balls. Exactly 12% are yellow, 20% are black. Is it possible that exactly 2/3 of the balls are white?

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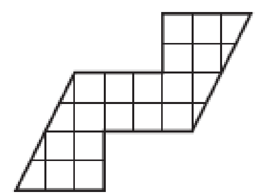
Problems
10-12

1. Twenty five horses participate in a competition. The competition consists of seven runs, five horse compete in each run. Each horse shows the same result in any run it takes part. No two horses will give the same result. After each run you can decide what horses participate in the next run. Could you determine the three fastest horses? (You don't have stopwatch. You can only remember the order of the horses.)

2. Prove that the equation $x^6 - 143x^5 - 917x^4 + 51x^3 + 77x^2 + 291x + 1575 = 0$ does not have solutions in integer numbers.

3. Show how we can cut the figure shown in the picture into two parts for us to be able to assemble a square out of these two parts. Show how we can assemble a square.



4. The city of Vyatka in Russia produces local drink, called "Vyatka Cola". «Vyatka Cola» is sold in 1, 3/4, and 1/2-gallon bottles. Ivan and John bought 4 gallons of "Vyatka Cola". Can we say **for sure**, that they can split the Cola evenly between them without opening the bottles?

5. Positive numbers a, b and c satisfy the condition $a + bc = (a + b)(a + c)$. Prove that $b + ac = (b + a)(b + c)$.