2019 MID-MICHIGAN MATHEMATICS OLYMPIAD:
GRADES 5-6

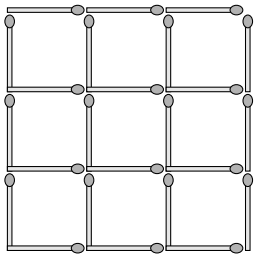
You must explain your answer. The points awarded are not just for correct answer, but also for your explanation.

PROBLEMS

(1) It takes 12 months for Santa Claus to pack gifts. It would take 20 months for his apprentice to do the job. If they work together, how long will it take for them to pack the gifts?

(2) All passengers on a bus sit in pairs. Exactly 2/5 of all men sit with women, exactly 2/3 of all women sit with men. What part of passengers are men?

(3) There are 100 colored balls in a box. Every 10-tuple of balls contains at least two balls of the same color. Show that there are at least 12 balls of the same color in the box.

(4) There are 81 wheels in storage marked by their two types, say first and second type. Wheels of the same type weigh equally. Any wheel of the second type is much lighter than a wheel of the first type. It is known that exactly one wheel is marked incorrectly. Show that one can determine which wheel is incorrectly marked with four measurements.

(5) Remove from the figure below the specified number of matches so that there are exactly 5 squares of equal size left: (a) 8 matches (b) 4 matches