

**2019 MID-MICHIGAN MATHEMATICS OLYMPIAD:  
GRADES 10-12**

**You must explain your answer.** The points awarded are not just for correct answer, but also for your explanation.

**PROBLEMS**

- (1) In triangle  $ABC$ , the median  $BM$  is drawn. The length  $|BM| = |AB|/2$ . The angle  $\angle ABM = 50^\circ$ . Find the angle  $\angle ABC$ .

*Solution: Extend median  $BM$  beyond point  $M$  and find point  $K$  on the ray  $BM$  such that  $|KM| = |MB|$ . Since,  $|AB| = 2|BM| = |AK|$  the triangle  $ABK$  is isosceles. Hence, angles  $BAK$  and  $BKA$  are equal to  $(180^\circ - 50^\circ)/2 = 65^\circ$  each. Note that  $ABCK$  is a parallelogram since its diagonal are split in halves by their intersection point. Therefore,  $\angle ABC = 50^\circ + 65^\circ = 115^\circ$ .*

- (2) Is there a positive integer  $n$  which is divisible by each of  $1, 2, 3, \dots, 2018$  except for two numbers whose difference is 7?

*Solution: answer is NO. If such  $n$  exists then the both numbers that are not divisors of  $n$  must be primes. Hence, there are two primes whose difference is 7. Such primes do not exist. Indeed, otherwise one of the primes must be equal to 2. Hence the second one is 9 which is not prime.*

- (3) Twenty numbers are placed around the circle in such a way that any number is the average of its two neighbors. Prove that all of the numbers are equal.

- (4) A finite number of frogs occupy distinct integer points on the real line. At each turn, a single frog jumps by 1 to the right so that all frogs again occupy distinct points. For some initial configuration, the frogs can make  $n$  moves in  $m$  ways. Prove that if they jump by 1 to the left (instead of right) then the number of ways to make  $n$  moves is also  $m$ .

- (5) A square box of chocolates is divided into 49 equal square cells, each containing either dark or white chocolate. At each move Alex eats two chocolates of the same kind if they are in adjacent cells (sharing a side or a vertex). What is the maximal number of chocolates Alex can eat regardless of distribution of chocolates in the box?