## MidMichigan Mathematical Olympiad, Spring 2022

## Grades 5-6

1. An animal farm has geese and pigs with a total of 30 heads and 84 legs. Find the number of pigs and geese on this farm.
2. What is the maximum number of $1 \times 1$ squares of a $7 \times 7$ board that can be colored black in such a way that the black squares don't touch each other even at their corners? Show your answer on the figure below and explain why it is not possible to get more black squares satisfying the given conditions.

3. Decide whether it is possible to divide a regular hexagon into three equal not necessarily regular hexagons? A regular hexagon is a hexagon with equal sides and equal angles.

4. A rectangle is subdivided into a number of smaller rectangles. One observes that perimeters of all smaller rectangles are whole numbers. Is it possible that the perimeter of the original rectangle is not a whole number?
5. Place parentheses on the left hand side of the following equality to make it correct.

$$
4 \times 12+18 \div 6+3=50
$$

6. Is it possible to cut a $16 \times 9$ rectangle into two equal parts which can be assembled into a square?

## MidMichigan Mathematical Olympiad, Spring 2022 <br> GRADES 7-9

1. Find the unknown angle $\alpha$ of the triangle inscribed in the square.

2. Draw a polygon in the plane and a point outside of it with the following property: no edge of the polygon is completely visible from that point (in other words, the view is obstructed by some other edge).
3. This problem has two parts. In each part, 2022 real numbers are given, with some additional property.
(a) Suppose that the sum of any three of the given numbers is an integer. Show that the total sum of the 2022 numbers is also an integer.
(b) Suppose that the sum of any five of the given numbers is an integer. Show that 5 times the total sum of the 2022 numbers is also an integer, but the sum itself is not necessarily an integer.
4. Replace stars with digits so that the long multiplication in the example below is correct.

5. Five nodes of a square grid paper are marked (called marked points). Show that there are at least two marked points such that the middle point of the interval connecting them is also a node of the square grid paper.


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GRADES 10-12

1. Consider a triangular grid: nodes of the grid are painted black and white. At a single step you are allowed to change colors of all nodes situated on any straight line (with the slope $0^{\circ}$, $60^{\circ}$, or $120^{\circ}$ ) going through the nodes of the grid. Can you transform the combination in the left picture into the one in the right picture in a finite number of steps?

2. Find $x$ satisfying $\sqrt{x \sqrt{x \sqrt{x}}} \ldots=\sqrt{2022}$ where it is an infinite expression on the left side.
3. 179 glasses are placed upside down on a table. You are allowed to do the following moves. An integer number $k$ is fixed. In one move you are allowed to turn any $k$ glasses .
(a) Is it possible in a finite number of moves to turn all 179 glasses into "bottom-down" positions if $\mathrm{k}=3$ ?
(b) Is it possible to do it if $\mathrm{k}=4$ ?
4. An interval of length 1 is drawn on a paper. Using a compass and a simple ruler construct an interval of length $\sqrt{93}$
5. Show that $5^{2 n+1}+3^{n+2} 2^{n-1}$ is divisible by 19 for any positive integer $n$.
6. Solve the system

$$
\left\{\begin{array}{l}
\frac{x y}{x+y}=1-z \\
\frac{y z}{y+z}=2-x \\
\frac{z x}{z+x}=2-y
\end{array}\right.
$$

