MidMichigan Mathematical Olympiad 2023

Grades 5-6

1) Solve: **INK + INK + INK + INK + INK + INK = PEN**
   (INK and PEN are 3-digit numbers, and different letters stand for different digits).

2) Two people play a game. They put 3 piles of matches on the table:
   the first one contains 1 match, the second one 3 matches, and the third one 4 matches.
   Then they take turns making moves. In a move, a player may take any nonzero number of matches FROM ONE PILE. The player who takes the last match from the table loses the game.
   a) The player who makes the first move can win the game.
      What is the winning first move?
   b) How can he win? (Describe his strategy.)

3) The planet Naboo is under attack by the imperial forces. Three rebellion camps are located at the vertices of a triangle. The roads connecting the camps are along the sides of the triangle. The length of the first road is less than or equal to 20 miles, the length of the second road is less than or equal to 30 miles, and the length of the third road is less than or equal to 45 miles. The Rebels have to cover the area of this triangle with a defensive field.
   What is the maximal area that they may need to cover?

4) Money in Wonderland comes in \$5 and \$7 bills. What is the smallest amount of money you need to buy a slice of pizza that costs \$1 and get back your change in full? (The pizza man has plenty of \$5 and \$7 bills.) For example, having \$7 won't do, since the pizza man can only give you \$5 back.

5) (**a**) Put 5 points on the plane so that each 3 of them are vertices of an isosceles triangle (i.e., a triangle with two equal sides), and no three points lie on the same line.
   (**b**) Do the same with 6 points.

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Grades 7-9

1) Three camps are located in the vertices of an equilateral triangle. The roads connecting camps are along the sides of the triangle. Captain America is inside the triangle and he needs to know the distances between camps. Being able to see the roads he has found that the sum of the shortest distances from his location to the roads is 50 miles. Can you help Captain America to evaluate the distances between the camps?

2) N regions are located in the plane, every pair of them have a non-empty overlap. Each region is a connected set, that means every two points inside the region can be connected by a curve all points of which belong to the region. Iron Man has one charge remaining to make a laser shot. Is it possible for him to make the shot that goes through all N regions?

3) Money in Wonderland comes in \$5 and \$7 bills.
(a) What is the smallest amount of money you need to buy a slice of pizza that costs \$1 and get back your change in full? (The pizza man has plenty of \$5 and \$7 bills.) For example, having \$7 won't do since the pizza man can only give you \$5 back.
(b) Vending machines in Wonderland accept only exact payment (do not give back change). List all positive integer numbers which CANNOT be used as prices in such vending machines. (That is, find the sums of money that cannot be paid by exact change.)

4) (a) Put 5 points on the plane so that each 3 of them are vertices of an isosceles triangle (i.e., a triangle with two equal sides), and no three points lie on the same line.
(b) Do the same with 6 points.

5) Numbers 1,2,3,...,100 are randomly divided in two groups 50 numbers in each. In the first group the numbers are written in increasing order and denoted $a_1, a_2, \dots, a_{50}$. In the second group the numbers are written in decreasing order and denoted $b_1, b_2, \dots, b_{50}$. Thus, $a_1 < a_2 < \cdots < a_{50}$ and $b_1 > b_2 > \cdots > b_{50}$.
Evaluate $|a_1 - b_1| + |a_2 - b_2| + \cdots + |a_{50} - b_{50}|$.

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Grades 10-12

1. There are 16 students in a class. Each month the teacher divides the class into two groups. What is the minimum number of months that must pass for any two students to be in different groups in at least one of the months?

2. Find all functions f(x) defined for all real x that satisfy the equation $2f(x) + f(1 - x) = x^2$.

3. Arrange the digits from 1 to 9 in a row (each digit only once) so that every two consecutive digits form a two-digit number that is divisible by 7 or 13.

4. Prove that $\cos 1^o$ is irrational.

5. Consider 2n distinct positive Integers $a_1, a_2, \dots, a_{2n}$ not exceeding n^2 $(n > 2)$. Prove that some three of the differences $a_i - a_j$ are equal.