Name SOLUTIONS April 14, 2009

Math 133 — Quiz 10A

(1) (4 points) Find the degree 3 Taylor polynomial $P_3(x)$ of $f(x) = \dfrac{1}{\sqrt{x}}$ at $a = 1$.

n	$f^{(n)}(x)$	at $x=1$	
0	$\frac{1}{\sqrt{x}} = x^{-1/2}$	1	1 pt.
1	$-\frac{1}{2}x^{-3/2}$	$-\frac{1}{2}$	
2	$-\frac{1}{2}\cdot\frac{-3}{2}x^{-5/2}$	$\frac{3}{4}$	1 pt
3	$-\frac{1}{2}\cdot\frac{-3}{2}\cdot\frac{-5}{2}x^{-7/2}$	$-\frac{15}{8}$	1 pt

$$P_3(x) = 1 - \tfrac{1}{2}(x-1) + \tfrac{3}{4}(x-1)^2 - \tfrac{15}{8}(x-1)^3$$

1 pt.

(2) (4 points) Starting from a known series, write down the complete Maclaurin Series of $f(x) = x\cos\sqrt{x}$. *Show your reasoning.*

$$\cos u = 1 - \frac{u^2}{2!} + \frac{u^4}{4!} - \frac{u^6}{6!} + \cdots \qquad \text{1 pt.}$$

(+1) take $u = \sqrt{x}$

$$\cos\sqrt{x} = 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \cdots$$

(+1) multiply by x

$$x\cos\sqrt{x} = x - \frac{x^2}{2!} + \frac{x^3}{4!} - \frac{x^4}{6!} + \cdots \quad \text{or} \quad \sum_{n=0}^{\infty}(-1)^n\frac{x^{n+1}}{(2n)!}$$

Answer +1

either form is OK.

(3) (2 points) Write the first 4 terms in the Binomial Series for $(1+x)^{1/2}$.

$$(1+x)^{1/2} = 1 + \tfrac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}x^3 + \cdots$$

1 pt

$$= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8} + \cdots$$

1 pt