Name Solutions April 14, 2009

Math 133 — Quiz 10B

(1) (4 points) Find the degree 3 Taylor polynomial $P_3(x)$ of $f(x) = \arctan x$ at $a = 0$.

n	$f^{(n)}(x)$	at $x=0$
0	$\arctan x$	0
1	$\frac{1}{1+x^2}$	1
2	$\frac{-2x}{(1+x^2)^2}$	-2
3	$\frac{-2(1+x^2)^2+2x\cdot(\text{stuff})}{(1+x^2)^4}$	-2

1 pt 1 pt 1 pt

$$P_3(x) = 0 + 1\cdot x + \frac{(-2)}{2!}x^2 + \frac{(-2)}{3!}x^3$$

$$\boxed{P_3(x) = x - x^2 - \frac{x^3}{3}}$$

1 pt

(2) (4 points) Starting from a known series, write down the complete Maclaurin Series of $f(x) = xe^{-x^2/2}$. *Show your reasoning.*

$$e^u = 1 + u + \frac{u^2}{2!} + \frac{u^3}{3!} + \frac{u^4}{4!} + \cdots \quad (+1)$$

(+1) take $u = -x^2/2$

$$e^{-\frac{x^2}{2}} = 1 + \left(-\frac{x^2}{2}\right) + \frac{1}{2!}\left(\frac{-x^2}{2}\right)^2 + \frac{1}{3!}\left(\frac{-x^2}{2}\right)^3 + \frac{1}{4!}\left(\frac{-x^2}{2}\right)^4 + \cdots$$

$$= 1 - \frac{x^2}{2} + \frac{x^4}{2^2\cdot 2!} - \frac{x^6}{2^3\cdot 3!} + \frac{x^8}{2^4\cdot 4!} - \cdots$$

(+1) Multiply by x

$$xe^{-x^2/2} = x - \frac{x^3}{2} + \frac{x^5}{2\cdot 2!} - \frac{x^7}{2^3\cdot 3!} + \frac{x^9}{2^4\cdot 4!} - \cdots \quad \text{or} \quad \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2^n\cdot n!}$$

Correct Answer (1 pt) either form is OK

(3) (2 points) Write the first 4 terms in the Binomial Series for $(1 + x)^{-1/2}$.

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}x^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}x^3 + \cdots \quad (1\text{ pt})$$

$$= 1 - \frac{x}{2} + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \cdots \quad (1\text{ pt})$$