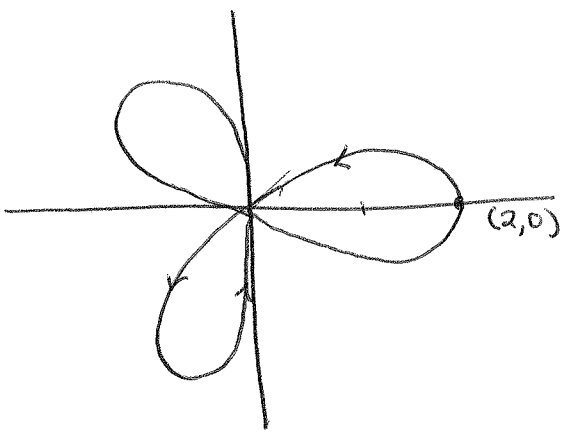
Name SOLUTIONS April 28, 2009

Math 133 — Quiz 11

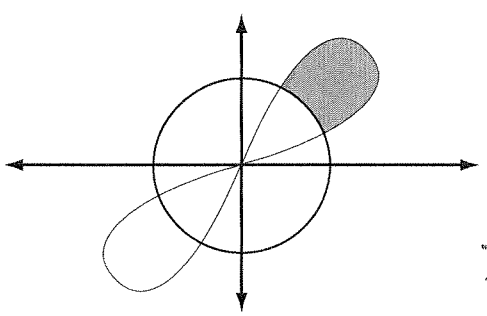
(1) (6 points) Graph the polar equation $r = 2\cos 3\theta$. Label the xy coordinates of one of the points that is furthest from the origin.

This is a 3-leafed rose

θ	r
0	2
$30^\circ = \pi/6$	0
$60^\circ = \pi/3$	-2



(2) (6 points) Find the area of the shaded region in the first quadrant outside the circle $r = 1$ and inside the lemniscate $r^2 = 2\sin(2\theta)$.



Intersect at $r = 1 = 2\sin(2\theta)$

$\frac{1}{2} = \sin(2\theta)$

$2\theta = 30^\circ$ or 150°

$= \pi/6$ or $5\pi/6$

$\theta = \pi/12$ or $5\pi/12$

Inner radius $r = 1$

Outer radius $R^2 = 2\sin(2\theta)$

$$\text{Area} = \int \tfrac{1}{2}(R^2 - r^2)\,d\theta = \int_{\pi/12}^{5\pi/12} \tfrac{1}{2}\left[2\sin(2\theta) - 1\right] d\theta$$

$$= -\cos(2\theta) - \frac{\theta}{2}\Big|_{\pi/12}^{5\pi/12}$$

$$= -\cos(150^\circ) + \cos 30^\circ - \frac{1}{2}\cdot\frac{4\pi}{12}$$

$$= -\left(\frac{-\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} - \pi/6$$

$$= \boxed{\sqrt{3} - \pi/6}$$