

Math 133 — Quiz 2A

- (1) (4 points) Write down an integral that gives the length of the part of the parabola $y = 4x - x^2$ that lies above the x -axis. Sketch, but do not evaluate the integral.



picture +1

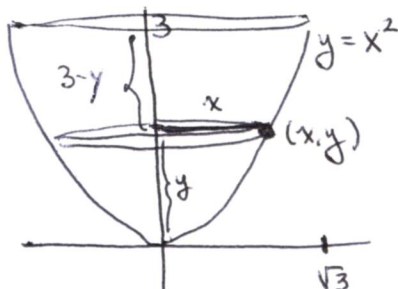
Intersection with x -axis: $0 = 4x - x^2 = x(4-x) \rightarrow x=0, 4$
+1

$$\begin{aligned} \text{Arc length} &= \int_0^4 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx. \quad \text{general form +1} \\ &= \int_0^4 \sqrt{1 + (4-2x)^2} dx. = \int_0^4 \sqrt{4x^2 - 16x + 17} dx \end{aligned}$$

Answer +1.

- (2) (8 points) Solve the following problem. Start with a sketch and make your reasoning clear.

A tank of water is constructed by revolving the curve $y = x^2$, $0 \leq x \leq \sqrt{3}$ around the y -axis, where the units are in meters. The water weighs 10000 N/m^3 . How much work does it take to empty the tank by pumping the water to the tank's top?



picture +1

Label slices by y $0 \leq y \leq 3$ Slice at y has:

$$\begin{aligned} \text{Volume} &: \pi x^2 \Delta y \quad (\text{radius} = x) \\ &= \pi y \Delta y \quad (y = x^2). \quad +1 \end{aligned}$$

$$\text{Weight} : 10000 \pi y \Delta y \quad +1$$

$$\text{Lift distance } (3-y) \quad +1$$

$$\text{Work} = \int_0^3 10,000 \pi y (3-y) dy \quad +1$$

$$= 10,000 \pi \int_0^3 3y - y^2 dy.$$

$$= 10,000 \pi \left[\frac{3}{2} y^2 - \frac{y^3}{3} \right]_0^3$$

$$= 10,000 \pi \left[\frac{27}{2} - 9 \right] = \boxed{45,000 \pi \text{ Newton-meters}}$$

+2