

Math 133 — Quiz 2B

(1) (4 points) Write down an integral that gives the length of the graph of $y = \tan x$ from $x = 0$ to $x = \frac{\pi}{4}$. Do not evaluate the integral.

$$y = \tan x$$

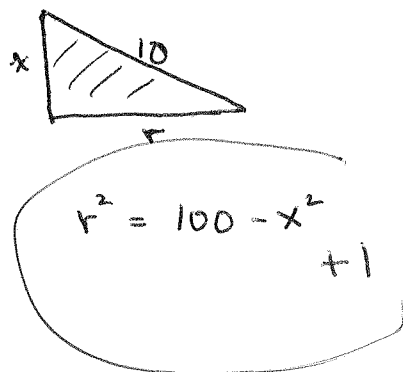
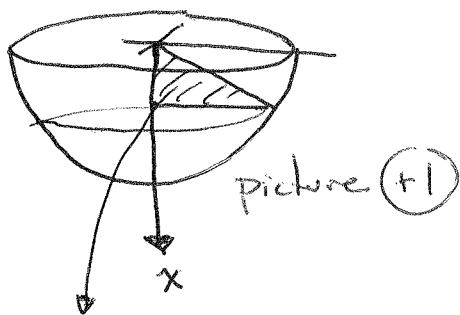
$$\frac{dy}{dx} = \sec^2 x$$

$$\text{Arc length} = \int_0^{\pi/4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad \text{general form +1}$$

$$= \int_0^{\pi/4} \sqrt{1 + \sec^4 x} dx \quad \text{answer +1}$$

(2) (8 points) Solve the following problem. Start with a sketch and make your reasoning clear.

A hemispherical tank with a radius of 10 feet is full of oil. The oil weighs 50 lbs/ft³. How much work does it take to empty the tank by pumping the oil to the tank's top?



Label slices with x as shown. $0 \leq x \leq 10$
 Slice at x has

Volume: $\pi r^2 \Delta x = \pi (100 - x^2) \Delta x$ (+1)

Weight: $50 \cdot \pi (100 - x^2) \Delta x$ (+1)

Lift distance: x (+1)

Work = $\int_0^{10} 50 \pi (100 - x^2) x dx$ (+1) limits (+1)

$$= 50 \pi \left[50x^2 - \frac{x^4}{4} \right]_0^{10} \quad (+1)$$

$$= 50 \pi \left[50 \cdot 100 - \frac{100 \cdot 100}{4} \right]$$

$$= 50 \pi \cdot 25 \cdot 100 = 50 \cdot 2500 \pi \text{ ft-lbs}$$

or 125,000 ft-lbs.