Name SOLUTIONS

Jan 27, 2009

Math 133 — Quiz 3A

(1) (3 points) Find $\frac{dy}{ds}$ when $y = \ln(x^2+1)$.

This is $y = \ln(u)$ where $u = x^2+1$. Hence $\frac{dy}{dx} = \frac{1}{u}\frac{du}{dx} = \boxed{\frac{2x}{x^2+1}}$

3 pts

(2) (3 points) If $w(s) = \sec\left(e^{-s^2}\right)$, find $\frac{dw}{ds}$.

Write $w = \sec(u)$.

$$\frac{dw}{ds} = \overbrace{\sec(u)\cdot\tan(u)}^{+1}\cdot\frac{du}{ds}$$

$$= \sec(e^{-s^2})\cdot\tan(e^{-s^2})\cdot e^{-s^2}\cdot(-2s)$$

+1

+1

(3) (3 points) If $y = \sqrt{\frac{(x+1)^8}{(2x+3)^3}}$, express $\frac{dy}{dx}$ in terms of *both* x and y. *Hint*: take ln of both sides, then differentiate.

Take ln:

$$\ln y = \frac{1}{2}\ln\left(\frac{(x+1)^8}{(2x+3)^3}\right) = \frac{1}{2}\left[\underbrace{8\ln(x+1) - 3\ln(2x+3)}_{+1}\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{4}{x+1} - \frac{3\cdot\cancel{2}}{\cancel{2}(2x+3)} \quad +1$$

$$\boxed{\frac{dy}{dx} = y\left(\frac{4}{x+1} - \frac{3}{2x+3}\right)} \quad +1$$

(4) (3 points) Find $\int \frac{\sec^2 t\, dt}{6+3\tan t}$.

Set $u = 6+3\tan t$
$du = 3\sec^2 t\, dt.$
$\frac{1}{3}du = \sec^2 t\, dt$

+1

$$\int\frac{\sec^2 t\, dt}{6+3\tan t} = \frac{1}{3}\int\frac{du}{u} \quad +1$$

$$= \frac{1}{3}\ln|u| + C$$

$$= \frac{1}{3}\ln|6+3\tan t| + C$$

+1

no absolute value: $-\frac{1}{2}$.