Name SOLUTIONS Jan 27, 2009

Math 133 — Quiz 3B

(1) (3 points) Differentiate $y = e^{-x^2}$.

This is $y = e^u$ where $u = -x^2$. Hence

$$\frac{dy}{dx} = e^u \frac{du}{dx} = e^{-x^2}(-2x) = -2x\,e^{-x^2}$$

3 pts

(2) (3 points) Find $\frac{dw}{dt}$ when $w(t) = \ln\left(\frac{e^t}{1+e^t}\right)$. *Hint:* Simplify first!

Simplify:

$$w = \ln e^t - \ln(1+e^t)$$
$$= t - \ln(1+e^t)$$
$$\frac{dw}{dt} = \boxed{1 - \frac{e^t}{1+e^t}} \quad \left[\text{or } = \frac{1+e^t}{1+e^t} - \frac{e^t}{1+e^t} = \frac{1}{1+e^t}\right]$$

3 pts

(3) (3 points) Differentiate $\ln y = e^y \tan x$ implicitly to express y' as a function of x and y.

$\frac{1}{y}y' = e^y y' \cdot \tan x + e^y \cdot \sec^2 x$ (+1) (+1) (by product rule)

$$\left(\frac{1}{y} - e^y \tan x\right) y' = e^y \sec^2 x$$

$$\boxed{y' = \frac{e^y \sec^2 x}{\frac{1}{y} - e^y \tan x}} \quad (+1)$$

(4) (3 points) Find $\int \frac{\sec\theta \tan\theta}{2+\sec\theta}\,d\theta$.

[Set $u = 2 + \sec\theta$, $du = \sec\theta\tan\theta\,d\theta$.] (+1)

$$= \int \frac{du}{u}$$
$$= \ln|u| + C \quad (+1)$$
$$= \ln|2+\sec\theta| + C \quad (+1)$$

no absolute value (−½)