Name Solutions & Key                                                  Feb 10, 2009

# Math 133 — Quiz 4A

(1) (3 points) Compare the growth of the functions $\sqrt{x}$ and $\ln(2x+5)$ as $x \to \infty$. *Which grows faster?* Show your reasoning. If you use L'Hopital's rule, identify the type. **State your answer clearly.**

$$\lim_{x\to\infty} \frac{\sqrt{x}}{\ln(2x+5)} \overset{\text{L'Hop}}{=} \lim_{x\to\infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{2x+5}\cdot 2} = \lim_{x\to\infty} \frac{2x+5}{4\sqrt{x}}$$

type $\frac{\infty}{\infty}$

$-\frac{1}{2}$ if not stated

1 pt

$$= \lim_{x\to\infty} \frac{1}{4}\left(\frac{2x}{\sqrt{x}} + \frac{5}{\sqrt{x}}\right)$$

$$= \frac{1}{4} \lim_{x\to\infty} \left(2\sqrt{x} + \frac{5}{\sqrt{x}}\right) = \infty$$

+1 pt

$\Rightarrow$ $\sqrt{x}$ grows faster than $\ln(2x+5)$

Conclusion 1 pt

(2) (3 points) Find $\frac{d}{dx}\, 5\ln(\operatorname{arcsec} x)$

$$= \underbrace{\frac{5}{\operatorname{arcsec} x}}_{1\text{ pt}} \cdot \underbrace{\frac{1}{|x|\sqrt{x^2-1}}}_{2\text{ pts}}$$

$-1$ for no absol. value.

(3) (3 points) Find $\int \frac{8\,dx}{x^2-2x+2}$

$$= \int \frac{8\,dx}{(x-1)^2+1}$$

[Set $u = x-1$, $du = dx$

complete the square (+1)

substitute (+1)

$$= 8\int \frac{du}{u^2+1}$$

$$= 8\arctan(u) + C$$

do integral (+1)

$$= \underline{8\arctan(x-1) + C}$$

(4) (3 points) Evaluate $\int_0^1 \frac{4y\,dy}{\sqrt{4-y^4}}$

Set $u = y^2$
$du = 2y\,dy$.

do indefinite. (+1)

$$\int \frac{4y\,dy}{\sqrt{4-y^4}} = \int \frac{2\,du}{\sqrt{4-u^2}} = 2\arcsin\left(\frac{u}{2}\right) + C = 2\arcsin\left(\frac{y^2}{2}\right) + C$$

$$\Rightarrow \int_0^1 \frac{4y\,dy}{\sqrt{4-y^2}} = 2\arcsin\left(\frac{y^2}{2}\right)\Big|_0^1 = 2\left[\arcsin\left(\tfrac{1}{2}\right) - \arcsin(0)\right]$$

$$= 2\left[\frac{\pi}{6} - 0\right]$$

$$= \frac{\pi}{3}$$

evaluate +1