

Math 133 — Quiz 4A

(1) (3 points) Compare the growth of the functions \sqrt{x} and $\ln(2x+5)$ as $x \rightarrow \infty$. Which grows faster? Show your reasoning. If you use L'Hopital's rule, identify the type. State your answer clearly.

$$\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\ln(2x+5)} \stackrel{\text{L'Hop}}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{2x+5} \cdot 2} = \lim_{x \rightarrow \infty} \frac{2x+5}{4\sqrt{x}}$$

↑ type $\frac{\infty}{\infty}$

$$= \lim_{x \rightarrow \infty} \frac{1}{4} \left(\frac{2x}{\sqrt{x}} + \frac{5}{\sqrt{x}} \right)$$

$$= \frac{1}{4} \lim_{x \rightarrow \infty} \left(2\sqrt{x} + \frac{5}{\sqrt{x}} \right) = \infty$$

$\Rightarrow \sqrt{x}$ grows faster than $\ln(2x+5)$

Handwritten notes: -1/2 if not started (pointing to L'Hop), +1 pt (pointing to the final result), Conclusion 1 pt.

(2) (3 points) Find $\frac{d}{dx} 5 \ln(\operatorname{arcsec} x)$

$$= \frac{5}{\operatorname{arcsec} x} \cdot \frac{1}{|x|\sqrt{x^2-1}}$$

Handwritten notes: 1 pt (under arcsec x), 2 pts (under denominator), -1 for no abs. value.

(3) (3 points) Find $\int \frac{8 dx}{x^2 - 2x + 2}$

$$= \int \frac{8 dx}{(x-1)^2 + 1} \quad \left[\begin{array}{l} \text{Set } u = x-1 \\ du = dx \end{array} \right]$$

$$= 8 \int \frac{du}{u^2 + 1}$$

Handwritten notes: complete the square (+1), substitute (+1)

$$= 8 \operatorname{arctan}(u) + C$$

$$= \underline{8 \operatorname{arctan}(x-1) + C}$$

Handwritten notes: do integral (+1)

(4) (3 points) Evaluate $\int_0^1 \frac{4y dy}{\sqrt{4-y^4}}$

Set $u = y^2$
 $du = 2y dy$

do indefinite. (+1)

$$\int \frac{4y dy}{\sqrt{4-y^4}} = \int \frac{2 du}{\sqrt{4-u^2}} = 2 \operatorname{arcsin}\left(\frac{u}{2}\right) + C = 2 \operatorname{arcsin}\left(\frac{y^2}{2}\right) + C$$

$$\Rightarrow \int_0^1 \frac{4y dy}{\sqrt{4-y^4}} = 2 \operatorname{arcsin}\left(\frac{y^2}{2}\right) \Big|_0^1 = 2 \left[\operatorname{arcsin}\left(\frac{1}{2}\right) - \operatorname{arcsin}(0) \right]$$

$$= 2 \left[\frac{\pi}{6} - 0 \right]$$

$$= \left(\frac{\pi}{3} \right)$$

Handwritten notes: evaluate +1