Name Solutions & Key   Feb 10, 2009

Math 133 — Quiz 4B

(1) (3 points) Compare the growth of the functions $x^2 + 7x$ and $\ln(x)$ as $x \to \infty$. *Which grows faster?* Show your reasoning. If you use L'Hopital's rule, identify the type. **State your answer clearly.**

$$\lim_{x\to\infty} \frac{x^2+7x}{\ln x} \underset{\text{L'Hop}}{=} \lim_{x\to\infty} \frac{2x+7}{\frac{1}{x}} = \lim_{x\to\infty} x(2x+7) = \infty$$

type $\frac{\infty}{\infty}$ (1 pt)

1 pt

$-\frac{1}{2}$ if type not stated.

$\Rightarrow$ $x^2 + 7x$ grows faster than $\ln x$.

Conclusion 1 pt.

(2) (3 points) Find $\frac{d}{dx}\arctan(\ln x)$ $= \boxed{\dfrac{1}{1+\arctan^2(\ln x)} \cdot \dfrac{1}{x}}$

2 pts   1 pt.

(because $\frac{d}{dx}\arctan(u) = \frac{1}{1+u^2}\frac{du}{dx}$)

(3) (3 points) Find $\int \frac{dx}{x\sqrt{5x^2-4}}$

Method 1:

$$= \int \frac{dx}{x \cdot \sqrt{5}\sqrt{x^2 - 4/5}}$$

+1 factor out $\sqrt{5}$

do integral (+1)

$$= \frac{1}{\sqrt{5}} \frac{1}{\sqrt{\frac{4}{5}}} \operatorname{arcsec}\left|\frac{x}{\sqrt{4/5}}\right| + C$$

simplify (+1)

$$= \frac{1}{2} \operatorname{arcsec}\left|\frac{\sqrt{5}x}{2}\right| + C$$

-1 for no absol. value

or **Method 2:** Set $u = \sqrt{5}\cdot x$

$du = \sqrt{5}\,dx$

$\frac{1}{\sqrt{5}}du = dx$

$$= \int \frac{\frac{1}{\cancel{\sqrt{5}}}dx}{\frac{u}{\cancel{\sqrt{5}}}\cdot\sqrt{u^2-4}}$$

substitute (+1)

do integral (+2)

$$= \frac{1}{2}\operatorname{arcsec}\left|\frac{u}{2}\right| + C$$

$$= \frac{1}{2}\operatorname{arcsec}\left|\frac{\sqrt{5}x}{2}\right| + C$$

(4) (3 points) Evaluate $\int_0^{\pi/4} \frac{\sec^2 y\,dy}{\sqrt{1-\tan^2 y}}$

do indefinite integral (+1)

insert limits correctly (+1)

evaluate (+1)

Set $u = \tan y$

$du = \sec^2 y\,dy$.

When $y = 0$, u is $\tan(0) = 0$

$y = \frac{\pi}{4}$, u is $\tan(\frac{\pi}{4}) = 1$

$$= \int_0^1 \frac{du}{\sqrt{1-u^2}} = \arcsin(u)\Big|_0^1$$

$$= \arcsin(1) - \arcsin(0)$$

$$= \frac{\pi}{2} - 0 = \pi/2$$