Name Solution & Key

March 3, 2009

Math 133 — Quiz 6A

(1) (6 points) Find $\int \sin^4 x \cos^5 x \, dx$

Odd power of cos $\Rightarrow \left[\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array} \right]$. Then $\int \sin^4 x \cdot \cos^5 x \, dx$

$$= \int \sin^4 x \, (1 - \sin^2 x)^2 \cdot \cos x \, dx.$$

$$= \int u^4 (1-u^2)^2 \, du$$

1 point for each step

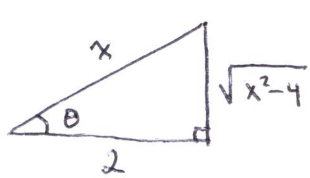
$$= \int u^4 (1 - 2u^2 + u^4) \, du$$

$$= \int u^4 - 2u^6 + u^8 \, du$$

$$= \frac{u^5}{5} - \frac{2}{7}u^7 + \frac{u^9}{9} + C = \underline{\underline{\frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + C}}$$

(2) (6 points) Find $\int \frac{\sqrt{x^2 - 4}}{x} dx$ by making a trig substitution, and doing the resulting trig integral.

2 pts



$$x = 2\sec\theta \qquad (1)$$

$$dx = 2\sec\theta \cdot \tan\theta \, d\theta$$

$$\sqrt{x^2 - 4} = 2\tan\theta. \qquad (2)$$

$$\int \frac{\sqrt{x^2-4}}{x} dx = \int \frac{2\tan\theta \cdot \cancel{2\sec\theta} \tan\theta \, d\theta}{\cancel{2\sec\theta}}$$ 1 pt

$$= 2\int \tan^2\theta \, d\theta$$

1 pt

$$= 2\int \sec^2\theta - 1 \, d\theta$$

1 pt

$$= 2\left[\tan\theta - \theta\right] + C$$

1 pt

use (2) and (1) above.

$$= \underline{\underline{\sqrt{x^2-4} - 2\,\mathrm{arcsec}\left(\tfrac{x}{2}\right) + C}}$$