Name SOLUTION & KEY

March 3, 2009

Math 133 — Quiz 6B

(1) (6 points) Find $\int \tan^3 x \sec^3 x \, dx$

Odd power of tan $\Rightarrow \int \tan^3 x \sec^3 x \, dx =$

$= \int (\sec^2 x - 1) \cdot \sec^2 x \cdot (\tan x \sec x \, dx)$ ↘ $u = \sec x$, $du = \sec x \cdot \tan x \, dx$

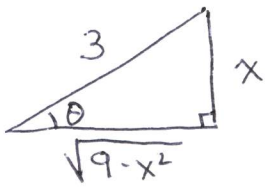
$= \int (u^2 - 1) u^2 \, du$

$= \int u^4 - u^2 \, du$

$= \frac{u^5}{5} - \frac{u^3}{3} + C = \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$

1 pt for each step

(2) (6 points) Find $\int \frac{x^2}{\sqrt{9-x^2}} \, dx$ by making a trig substitution, and doing the resulting trig integral.



$x = 3\sin\theta$ (1)

$dx = 3\cos\theta \, d\theta$

$\sqrt{9-x^2} = 3\cos\theta$ (2)

(1) (2) — 2 pts

$\int \frac{x^2 \, dx}{\sqrt{9-x^2}} = \int \frac{9\sin^2\theta \cdot 3\cos\theta \, d\theta}{3\cos\theta}$ — 1 pt

$= 9\int \sin^2\theta \, d\theta$ — 1 pt

$= \frac{9}{2}\int 1 - \cos(2\theta) \, d\theta$ — 1 pt

$= \frac{9}{2}\theta - \frac{9}{2}\cdot\frac{\sin(2\theta)}{2} + C$

$= \frac{9}{2}\theta - \frac{9}{2}\cdot\frac{2\sin\theta\cdot\cos\theta}{2} + C$ — 1 pt

use identities: $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\sin 2\theta = 2\sin\theta\cos\theta$

use (1) and (2) above.

$= \frac{9}{2}\left[\arcsin\left(\frac{x}{3}\right) - \frac{1}{9}x\sqrt{9-x^2}\right] + C$