Name SOLUTIONS March 17, 2009

Math 133 — Quiz 7A

(1) (4 points) Solve the differential equation $\sqrt{xy}\,\frac{dy}{dx} = 1$.

Separate variables and integrate: $\sqrt{y}\,dy = \frac{dx}{\sqrt{x}} \Rightarrow \int y^{1/2}\,dy = \int x^{-1/2}\,dx$

Separate variables +1

$\frac{2}{3} y^{3/2} = 2x^{1/2} + C$

integrate +2

so $y^{3/2} = \frac{3}{2}\left[2\sqrt{x} + C\right]$

$= 3\sqrt{x} + C$

new constant

solve for y +1

Thus $\boxed{y = (3\sqrt{x} + C)^{2/3}}$

(2) (4 points) Determine whether the following improper integral converges or diverges. Show your reasoning. (You may use either the Comparison Test or the Limit Comparison Test.)

$I = \int_1^\infty \frac{\sqrt{x}\,dx}{\sqrt{x^3+4}}$ For large x, $f(x) = \frac{\sqrt{x}}{\sqrt{x^3+4}}$ is approximately

$g(x) = \frac{\sqrt{x}}{\sqrt{x^3}} = \frac{\sqrt{x}}{x\sqrt{x}} = \frac{1}{x}$. In fact,

good comparison function +1

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{\sqrt{x}}{\sqrt{x^3+4}} \cdot \frac{x}{1} = \lim_{x\to\infty} \frac{\sqrt{x^3}}{\sqrt{x^3+4}} = \sqrt{\lim_{x\to\infty} \frac{x^3}{x^3+4}}$$

divide top & bottom by x^3

$$= \sqrt{\lim_{x\to\infty} \frac{1}{1+4/x^3}}$$

+1

$= 1$

+1

But $\int_1^\infty g(x)\,dx = \int_1^\infty \frac{1}{x}\,dx$ diverges, so the given integral I also diverges by the Limit Comparison Test.

Correct answer
Converges or diverges
+1 and reason