Name SOLUTIONS

March 17, 2009

Math 133 — Quiz 7B

(1) (4 points) Solve the differential equation $\frac{dy}{dx} = \sqrt{y}\cos^2(\sqrt{y})$. Separate variable & integrate:

Separate +1

$$\frac{1}{\sqrt{y}} \frac{1}{\cos^2\sqrt{y}}\, dy = dx.$$

$$\int \sec^2(\sqrt{y}) \frac{dy}{\sqrt{y}} = \int dx$$

$$\left[\begin{array}{l} u = \sqrt{y} = y^{1/2} \\ du = \frac{1}{2} y^{-1/2} dy = \frac{dy}{2\sqrt{y}}. \\ 2du = \frac{dy}{\sqrt{y}}. \end{array}\right.$$

Do integral +2

$$2\int \sec^2 u\, du = x + C$$

$$2 \tan u = x + C$$

Solve for y +1

Thus $\tan\sqrt{y} = \frac{x+C}{2} \Rightarrow \sqrt{y} = \arctan\left(\frac{x+C}{2}\right)$ so $\boxed{y = \left[\arctan\left(\frac{x+C}{2}\right)\right]^2}$

(2) (4 points) Determine whether the following improper integral converges or diverges. Show your reasoning. (You may use either the Comparison Test or the Limit Comparison Test.)

$$I = \int_1^\infty \frac{\sqrt{x-1}\, dx}{x^2}$$

+1

good comparison function +1

Method 1: Since $x - 1 < x$ we have $\frac{\sqrt{x-1}}{x^2} < \frac{\sqrt{x}}{x^2} = \frac{1}{x^{3/2}}$. But $\int_1^\infty \frac{dx}{x^{3/2}}$ converges (it's $\int_1^\infty \frac{dx}{x^p}$ with $p > 1$). Hence the given integral converges by the comparison test

Check comparison function +1

Method 2 For large x, $f(x) = \frac{\sqrt{x-1}}{x^2}$ is approximately $g(x) = \frac{\sqrt{x}}{x^2} = \frac{1}{x^{3/2}}$. Note

+1

a) $$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{\sqrt{x-1}}{x^2} \cdot \frac{x^{3/2}}{1} = \lim_{x\to\infty} \frac{\sqrt{x-1}}{\sqrt{x}}$$

$$= \sqrt{\lim_{x\to\infty} \frac{x-1}{x}} = \sqrt{\lim_{x\to\infty} 1 - \frac{1}{x}} = 1$$

b) $\int_1^\infty \frac{1}{x^{3/2}} dx$ is finite

Correct answer and reason +1

Hence I converges by the Limit Comparison Test.