Name SOLUTIONS March 31, 2009

Math 133 — Quiz 8A

(1) (4 points) Does the series $S = \sum_{n=0}^{\infty} \frac{-8}{3^n}$ converge or diverge? Give a reason. *If it converges, find its sum.*

$S = -8 \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$ geometric series (+1) with $r = \frac{1}{3} < 1 \Rightarrow$ converges to (+1)

$= -8 \frac{A}{1-r}$ (+1)

$= -8 \frac{1}{1-1/3} = -8 \cdot \frac{3}{2} = -12$ (+1)

(2) (4 points) Use the Integral Test to determine whether the series $S = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$ converges or diverges. Show your reasoning.

The function $f(x) = \frac{1}{\sqrt{x}(\sqrt{x}+1)}$ is positive for $x > 0$ (+1), is decreasing (+1) because the denominator $\sqrt{x}(\sqrt{x}+1)$ increases with x, and

$\int_1^{\infty} f(x)\,dx = \int_1^{\infty} \frac{dx}{\sqrt{x}(\sqrt{x}+1)}$

$\boxed{\begin{array}{ll} \text{let } u = \sqrt{x}+1 & \text{when } x = 1,\ u = 2 \\ du = dx/2\sqrt{x} & \text{when } x \to \infty \quad u \to \infty \end{array}}$

$= \int_2^{\infty} \frac{du}{u}$

$= \lim_{L\to\infty} \ln|u| \Big|_2^L = \lim_{L\to\infty} (\ln L) - \ln 2$ diverges (+1)

$\Rightarrow$ S diverges by the I Test. (+1)

(3) (4 points) Use the CT or LCT to determine whether the series $S = \sum_{n=1}^{\infty} \frac{1}{n\sqrt{3n-1}}$ converges or diverges. Show your reasoning.

Compare $a_n = \frac{1}{n\sqrt{3n-1}}$ to $\boxed{b_n = \frac{1}{n\sqrt{3n}} = \frac{1}{\sqrt{3}\, n^{3/2}}}$ (+1)

- $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1}{\cancel{n}\sqrt{3n-1}} \cdot \frac{\cancel{n}\sqrt{3n}}{1} = \sqrt{\lim_{n\to\infty} \frac{3n}{3n-1}}$ divide top & bottom by $3n$

  $= \sqrt{\lim_{n\to\infty} \frac{1}{1-1/3n}} = 1.$ (+1)

- $\sum_{n=1}^{\infty} b_n = \frac{1}{\sqrt{3}} \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges (p-series) (+1)

$\Rightarrow$ S converges by the LCT. (+1)