Name SOLUTIONS March 31, 2009

# Math 133 — Quiz 8B

(1) (4 points) Does the series $S = \sum_{n=0}^{\infty} \frac{-3}{2^n}$ converge or diverge? Give a reason. If it converges, find its sum.

$S = -3 \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ geometric series (+1) with $r = \frac{1}{2} < 1 \Rightarrow$ converges (+1) to

$= -3 \frac{A}{1-r}$ (+1)

$= -3 \frac{1}{1-1/2} = -3 \cdot 2 = \underline{\underline{-6}}$ (+1)

(2) (4 points) Use the Integral Test to determine whether the series $S = \sum_{n=1}^{\infty} \frac{\ln n}{n}$ converges or diverges. Show your reasoning.

The function $f(x) = \frac{\ln x}{x}$ is $\boxed{\text{positive for } x > 0}$ (+1), is decreasing for large $x$ since $f'(x) = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2} < 0$ for $x > e$ (+1) and

$\int_1^{\infty} f(x)\,dx = \int_1^{\infty} \frac{\ln x}{x}\,dx$ $\boxed{\begin{array}{ll} u = \ln x & \text{when } x = 1,\ u = 0 \\ du = \frac{dx}{x} & \text{when } x \to \infty,\ u \to \infty \end{array}}$

$= \int_0^{\infty} u\,du$

$= \lim_{L\to\infty} \frac{u^2}{2}\Big|_0^L = \frac{1}{2} \lim_{L\to\infty} L^2$ diverges (+1)

$\Rightarrow$ S diverges by the Integral Test. (+1)

(3) (4 points) Use the CT or LCT to determine whether the series $S = \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n(n+5)}$ converges or diverges. Show your reasoning.

Compare $a_n = \frac{\sqrt{n}}{n(n+5)}$ with $\boxed{b_n = \frac{\sqrt{n}}{n^2} = \frac{1}{n^{3/2}}}$ (+1)

Method 1 Since $n+5 > n$, $a_n = \frac{\sqrt{n}}{n(n+5)} < \frac{\sqrt{n}}{n^2} = b_n$ (+1). Also $\sum_{n=1}^{\infty} b_n = \sum \frac{1}{n^{3/2}}$ converges (p-series). (+1) $\Rightarrow$ S converges by the CT. (+1) (3)

Method 2. $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\cancel{\sqrt{n}}}{\cancel{n}(n+5)} \cdot \frac{n^{\cancel{2}}}{\cancel{\sqrt{n}}} = \lim_{n\to\infty} \frac{n}{n+5} = \lim_{n\to\infty} \frac{1}{1+5/n} = 1$ (+1)

Also, $\sum_{n=1}^{\infty} b_n = \sum \frac{1}{n^{3/2}}$ converges (p-series) (+1)

$\Rightarrow$ S converges by the LCT (+1) (3)