Name Solutions April 7, 2009

Math 133 — Quiz 9A

(1) (6 points – HW 11.6 #3) For which values of x does the power series $\sum_{n=1}^{\infty}(-1)^n(4x+1)^n$ converge absolutely? Show your reasoning. *Be sure to check the endpoints.*

$S = \sum_{n=1}^{\infty} \left(-(4x+1)\right)^n$ is a geometric series with $r = -(4x+1)$, so (+2)

converges absolutely for $|4x+1| < 1$ (+1) OR get here by Ratio Test (+3)

$-1 < 4x+1 < 1$

$-2 < 4x < 0$ (+1)

$-\tfrac{1}{2} < x < 0$ ← interval of convergence

and S diverges for $|4x+1| > 1$.

Endpoints: When $x=0$, $S = \sum (-1)^n$ diverges (+1)

When $x = -\tfrac{1}{2}$ $S = \sum (-1)^n(-1)^n = \sum_{n=1}^{\infty} 1$ diverges. (+1)

(2) (6 points – HW 11.6 #17) Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{n\,(x+3)^n}{5^n}$. Show your reasoning. *Do not check the endpoints.*

Apply the ratio test with $a_n = \frac{n\,|x+3|^n}{5^n}$, $a_{n+1} = \frac{(n+1)\,|x+3|^{n+1}}{5^{n+1}}$ (+1)

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)\cancel{|x+3|^n}\cdot|x+3|}{\cancel{5^n}\cdot 5}\cdot\frac{\cancel{5^n}}{n\,\cancel{|x+3|^n}}$$

Set up (+1)
Compute limit (+2)

$$= \frac{|x+3|}{5}\cdot\lim_{n\to\infty}\frac{n+1}{n}$$

$$= \frac{|x+3|}{5}\lim_{n\to\infty}\frac{1+1/n}{1} = \frac{|x+3|}{5}$$

+1

$\Rightarrow$ $\boxed{S \text{ converges absolutely when } \frac{|x+3|}{5} < 1}$ ~~, ie.~~ $|x+3| < 5$, so the interval of convergence is $-5 < x+3 < 5$

$\boxed{-8 < x < 2}$ (+1)