Name Solutions April 7, 2009

Math 133 — Quiz 9B

(1) (6 points – HW 11.6 #5) For which values of x does the power series $\sum_{n=1}^{\infty} \frac{(x-2)^n}{10^n}$ converge absolutely? Show your reasoning. *Be sure to check the endpoints.*

$S = \sum_{n=1}^{\infty} \left[\frac{(x-2)}{10}\right]^n$ is a geometric series with $r = \frac{x-2}{10}$, so converges (+2)

absolutely for $\left|\frac{x-2}{10}\right| < 1 \Rightarrow |x-2| < 10$ (+1) ← (or get here by Ratio Test +3)

(+1) $-10 < x-2 < 10$

$\underline{-8 < x < 12}$ ← interval of convergence.

S diverges for $|x-2| > 10$

Endpts: When $x = 12$, $S = \sum \frac{10^n}{10^n} = \sum_{n=1}^{\infty} 1$ diverges. (+1)

When $x = -8$, $S = \sum \frac{(-10)^n}{10^n} = \sum (-1)^n$ diverges. (+1)

(2) (6 points – HW 11.6 #29) Find the interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{3/2}}$. Show your reasoning. *Do not check the endpoints.*

Apply the Ratio Test (+1) with $a_n = \frac{|4x-5|^{2n+1}}{n^{3/2}}$ $a_{n+1} = \frac{|4x-5|^{2(n+1)+1}}{(n+1)^{3/2}}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{|4x-5|^{2n+3}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{|4x-5|^{2n+1}}$$

Set up (+1)

Compute limit (+2)

$$= \lim_{n\to\infty} |4x-5|^2 \cdot \frac{n^{3/2}}{(n+1)^{3/2}}$$

$$= |4x-5|^2 \cdot \left[\lim_{n\to\infty} \frac{n}{n+1}\right]^{3/2} = |4x-5|$$

$\Rightarrow$ The interval of convergence is $|4x-5| < 1$ (+1)

$-1 < 4x-5 < 1$

$4 < 4x < 6$

$\boxed{1 < x < 3/2}$. (+1)