

Instructions: Please show all of your work. Credit will not be given for answers with no supporting work.

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1. (18 pts) For each rational function, determine the correct form of the partial fractions decomposition, but **do not solve for the constants.**

(a)  $\frac{2x+1}{(x+1)(x-2)}$

(b)  $\frac{x^2+5x+1}{(x-1)^2(x^2+1)}$

(c)  $\frac{1}{x(x^2+4)^2}$

2. (21 pts) Compute  $\frac{dy}{dx}$ .

(a)  $y = \ln(\tan^{-1}(x^2))$

(b)  $y = \sec^{-1}\left(\frac{1}{x}\right)$

(c)  $y = \sqrt{\sinh(3x)}$

3. (36 pts) Calculate the following integrals.

(a)  $\int \frac{dx}{x^2 + 4x + 13}$

(b)  $\int_0^{\frac{\pi}{2}} x^2 \cos x \, dx$

(c)  $\int x \sec^{-1} x \, dx$  (assume  $x > 0$ )

(d)  $\int \sin^2 3x \, dx$

4. (15 pts) Given the partial fractions decomposition

$$\frac{3x^2 - 4x + 4}{(x+2)(x-2)(x-4)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C}{x-4}$$

(a) Compute the coefficients  $A$ ,  $B$  and  $C$ .

(b) Compute  $\int \frac{3x^2 - 4x + 4}{(x+2)(x-2)(x-4)} dx$

5. (10 pts) Compare the growth rates as  $x \rightarrow \infty$  of  $f(x) = x^2 \ln x$  and  $g(x) = x \ln(x^2 + 1)$ .