

Instructions: Please show all of your work. Credit will not be given for answers with no supporting work.

1.(20 pts) Compute each of the improper integrals.

(a)
$$\int_2^{\infty} \frac{dx}{x(\ln x)^2}$$

(b)
$$\int_0^2 \frac{x dx}{\sqrt{4-x^2}}$$

2.(20 pts) Solve the initial value problem. Write y explicitly as a function of x.

(a)
$$\frac{dy}{dx} = 1 - 2y \quad ; \quad y(0) = 0$$

(b)
$$\frac{dy}{dx} = \frac{x}{y} \quad ; \quad y(0) = 1$$

3.(24 pts) Classify each sequence as either convergent or divergent. If convergent, find the limit; if divergent, give reasons why.

(a) $a_n = \sqrt{\frac{2n^2}{n^2 + 1}}$

(b) $a_n = ne^{-n}$

(c) $a_n = \frac{\sqrt{n}}{\ln n}$

4.(16 pts) Classify each series as convergent or divergent. If convergent, find the sum; if divergent, give reasons why.

(a)
$$\sum_{k=1}^{\infty} 3\left(\frac{4}{7}\right)^k$$

(b)
$$\sum_{k=1}^{\infty} (-1)^k \left(\frac{k}{2k+1}\right)$$

5.(20 pts) Compute each of the integrals.

(a)
$$\int_0^1 \frac{dx}{(4-x^2)^{3/2}}$$

(b)
$$\int \frac{xdx}{\sqrt{4+x^2}}$$