Name SOLUTIONS Feb 3, 2009

Math 133 — First Hourly Exam B

Do all 5 problems (Total = 100 points). Show all work. Calculators or other electronic devices are not allowed.

(1) (27 points) Compute the following derivatives.

(a) $\frac{d}{d\theta}\ln(\sec\theta)$ $= \frac{1}{\sec\theta}(\sec\theta)' = \frac{\cancel{\sec\theta}\cdot\tan\theta}{\cancel{\sec\theta}} = \tan\theta$

(b) $\frac{d}{dx}\left(e^{-x}\log_{10}x^2\right)$ $= \frac{d}{dx}\left(e^{-x}\cdot 2\log_{10}x\right)$

$= 2(e^{-x})'\log_{10}x + 2e^{-x}(\log_{10}x)'$ (product rule)

$= -2e^{-x}\log_{10}x + 2e^{-x}\cdot\frac{1}{\ln 10}\cdot\frac{1}{x}.$

$= \underline{2e^{-x}\left[\frac{1}{x\ln 10} - \log_{10}x\right]}$

(c) Differentiate $y = (x^2+1)^{\sin x}$. Start by taking ln of both sides.

$\ln y = \sin x\cdot\ln(x^2+1)$

$\frac{1}{y}\frac{dy}{dx} = \cos x\cdot\ln(x^2+1) + \sin x\cdot\frac{2x}{x^2+1}$ product rule

$\frac{dy}{dx} = y\left[\cos x\cdot\ln(x^2+1) + \sin x\cdot\frac{2x}{x^2+1}\right]$

$\hookrightarrow$ with $y = (x^2+1)^{\sin x}$.

(2) (27 points) Compute the following integrals. Make your substitutions clear.

(a) $\displaystyle\int \frac{e^x}{3+2e^x}\,dx.$

Set $\left|\begin{array}{l} u = 3+2e^x \\ du = 2e^x\,dx. \\ \frac{1}{2}du = e^x\,dx \end{array}\right.$

$$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \underline{\frac{1}{2}\ln(3+2e^x) + C}$$

(b) $\displaystyle\int \frac{1}{t(\ln t)^2}\,dt.$

Set $\left|\begin{array}{l} u = \ln t \\ du = \frac{1}{t}\,dt \end{array}\right.$

$$= \int \frac{du}{u^2} = \int u^{-2}\,du = \frac{u^{-1}}{-1} + C = \underline{-\frac{1}{\ln t} + C}$$

(c) $\displaystyle\int_0^{\ln 2} \frac{1}{e^{2x}}\,dx.$

$$= \int_0^{\ln 2} e^{-2x}\,dx$$

$$= -\frac{1}{2}e^{-2x}\Big|_0^{\ln 2}$$

$$= -\frac{1}{2}\left(e^{-2\ln 2} - e^{-0}\right)$$

$$= -\frac{1}{2}\left((e^{\ln 2})^{-2} - 1\right)$$

$$= -\frac{1}{2}\left(2^{-2} - 1\right) = -\frac{1}{2}\left(\frac{1}{4} - 1\right) = -\frac{1}{2}\left(-\frac{3}{4}\right)$$

$$= \frac{3}{8}$$

(3) (10 points) At particle moves in the plane. Its position at time t is given by $x(t) = \cos t$ and $y(t) = \sqrt{t}$. Set up but **DO NOT EVALUATE** an integral that gives the distance the particle travels between $t = 0$ and $t = \pi/2$.

$$\text{Arclength} = \int_0^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{\sin^2 t + \frac{1}{4t}}\, dt$$

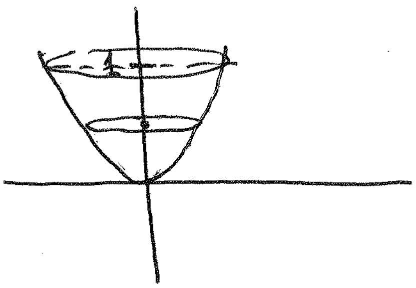
$x = \cos t$

$x' = -\sin t$

$y = \sqrt{t} = t^{1/2}$

$y' = \frac{1}{2} t^{-1/2} = \frac{1}{2\sqrt{t}}$

(4) (20 points) Let R be the region in the plane bounded by the x-axis, the line $y = 1$ and the curve $y = 9x^4$. Find the volume of the solid obtained by rotating R around the y-axis.



Slice at ht y is disk with area

$\pi r^2 = \pi x^2$

$= \frac{\pi}{3}\sqrt{y}.$

$y = 9x^4$

$\frac{1}{9} y = x^4$

$\frac{1}{3}\sqrt{y} = x^2$

$$\text{Volume} = \int_0^1 \frac{\pi}{3}\sqrt{y}\, dy$$

$$= \frac{\pi}{3} \int_0^1 y^{1/2}\, dy.$$

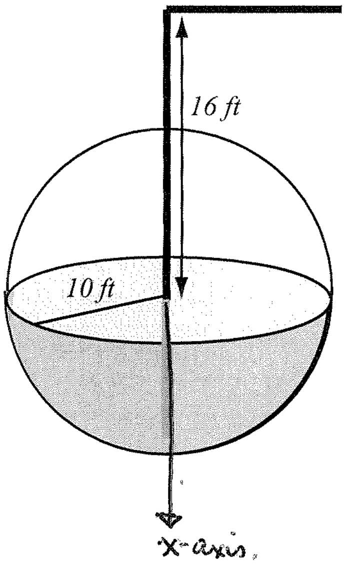
$$= \frac{\pi}{3} \frac{2}{3} y^{3/2}\Big|_0^1$$

$$= \frac{2\pi}{9}(1-0) = \frac{2\pi}{9}$$

typo! This should say 18 pts

(5) (~~28~~ points) A spherical water tank is 10 feet in ~~diameter~~ radius. It is exactly half full of water weighing 62.4 lbs per cubic foot. Set up but **Do NOT EVALUATE** a definite integral that gives the work needed to pump the water out of a pipe at the level 6 feet above the top of the tank, as shown. *Express your answer as a product of 62.4, π and some other numbers, and include units.*

Suggestion: Use the variable $x=$ distance down from the center of the tank.



Label slices by distance down from center of tank.

[top slice at $x=0$
[bottom slice at $x=10$

Slice at x is disk with Area

$\pi r^2 = \pi(100-x^2)$ by Pythag. Thm

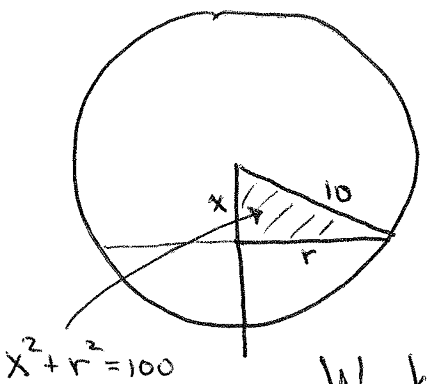
$\Rightarrow$ weight = density $\cdot$ volume

$= 62.4 \cdot \pi(100-x^2)\Delta x.$

Slice at x is lifted a distance of

$x+16$

(lift x feet to center of tank, then 16 feet more).



$$\text{Work} = \int \text{weight} \cdot \text{distance}$$

$$= 62.4\pi \int_0^{10} (100-x^2)(x+16)\,dx. \quad \text{foot-lbs.}$$