

Practice Problems for Math 133 Exam 3

1. Work out the following integrals (solutions shown).

$$(a) \int \sin^4 x \, dx = \frac{3x}{8} - \frac{\sin 2x}{4} + \frac{\sin 4x}{32} + C$$

$$(b) \int \tan^3 x \, dx = \frac{1}{2} \tan^2 x + \ln |\cos x| + C$$

$$(c) \int \frac{\sin^2 t \, dt}{\cos^4 t} = \frac{1}{3} \tan^3 t + C$$

$$(d) \int \tan^5 x \sec^3 x \, dx = \frac{\sec^7 x}{7} - \frac{2 \sec^5 x}{5} + \frac{\sec^3 x}{3} + C$$

$$(e) \int \sin^4 x \cos^2 x \, dx = \frac{x}{16} - \frac{\sin 4x}{64} - \frac{\sin^3 4x}{48} + C$$

2. Work out the following integrals using a trig substitution (solutions shown).

$$(a) \int \frac{x^2 \, dx}{\sqrt{x^2 - 6}} = \frac{x}{2} \sqrt{x^2 - 6} + 3 \ln \left| x + \sqrt{x^2 - 6} \right| + C$$

$$(b) \int \frac{t^2 \, dt}{\sqrt{4 - t^2}} = -\frac{t}{2} \sqrt{4 - t^2} + 2 \arcsin \frac{t}{2} + C$$

$$(c) \int \frac{dx}{x^3 \sqrt{x^2 - 9}} = \frac{\sqrt{x^2 - 9}}{18x^2} + \frac{1}{54} \operatorname{arcsec} \frac{x}{3} + C$$

$$(d) \int_2^4 \frac{\sqrt{16 - s^2}}{s^2} \, ds = \sqrt{3} - \frac{\pi}{3}$$

3. Work out the following improper integrals. Show all limits and justify your responses using the Comparison Test (CT) and Limit Comparison Test (LCT).

$$(a) \int_0^2 \frac{dx}{\sqrt{4 - x^2}} = \frac{\pi}{2}$$

$$(c) \int_2^\infty \frac{dx}{\sqrt[3]{x^4 - 1}} \text{ converges.}$$

$$(b) \int_0^{12} \frac{2x \, dx}{(x^2 - 16)^{2/3}} = 9 \cdot 16^{2/3}$$

$$(d) \int_{10}^\infty \ln(\ln x) \, dx \text{ diverges. (compare to } g(x) = 1)$$

(e) $\int_2^{\infty} \frac{dx}{x \ln x}$ diverges (note $\ln x \leq x$ for $x \geq 1$). (h) $\int_{-1}^1 -x \ln |x| dx$ diverges.

(f) $\int_0^{\infty} \frac{x dx}{(x^2 + 4)^{3/2}} = \frac{1}{6}$. (i) $\int_0^5 \frac{1 + \sin x}{(x - 2)} dx$ diverges.

(g) $\int_1^{\infty} \frac{dx}{\sqrt{x}(\sqrt{x} + 1)}$ diverges. (j) $\int_2^4 \frac{\cos x dx}{x\sqrt{x^2 - 4}}$ converges.

4. Find the general solution of the differential equation. When an initial condition is given, find the solution that satisfies the initial condition.

(a) $\frac{dy}{dx} = -\frac{\cos y}{\sin x}$ (c) $3(y^2 + 2) = 4y(x - 1) \frac{dy}{dx}$

(b) $\frac{dy}{dx} = 2x(1 + y^2), y(2) = 4$. (d) $\frac{dy}{dx} = \frac{e^{2x-y}}{e^{x+y}}, y(0) = \frac{1}{2}$

(e) In textbook, page 682: 3, 11, 25.

5. Determine if the following sequences converge or diverge. If the sequence converges find the limit. Justify your responses.

(a) $a_n = \frac{n + (-1)^n}{n}$

(d) $a_n = \frac{3^n}{n^3}$

(b) $a_n = \sqrt{\frac{n}{3n + 1}}$

(e) $a_n = \sqrt[n]{n^5}$

(f) $a_n = (n + 3)^{1/(n+4)}$

(c) $a_n = \frac{1}{(0.99)^n}$

(g) $a_n = \frac{\ln n}{n^c}, c > 0$

6. Determine if the following sequences converge or diverge. If the sequence converges find the limit. Again, justify your responses.

(a) $a_n = \frac{1}{n^{1+1/n}}$

(e) $a_n = \left(\frac{4n + 5}{4n - 1}\right)^n$

(b) $a_n = \ln n - \ln(n + 1)$

(f) $a_n = \frac{n!}{3^n 5^n}$

(c) $a_n = \left(1 + \frac{1}{n^2}\right)^n$

(g) $a_n = n - \sqrt{n^2 - n}$

(d) $a_n = \frac{(\ln n)^4}{\sqrt{n}}$

7. Determine whether the following geometric series converge or diverge. If the series converges find the limit. Again, justify your responses. For the repeating decimals, first write as a geometric series.

(a) $.1\overline{34}$

(b) $4.52\overline{159}$

(c) $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots$

(d) $\sum_{n=1}^{\infty} \frac{\pi^n}{e^n}$

(e) $\sum_{n=1}^{\infty} \frac{e^n}{\pi^n}$

(f) $\sum_{k=1}^{\infty} \frac{(-2)^k}{5^k}$

(g) $\sum_{n=1}^{\infty} \frac{6}{5^n} - \left(\frac{1}{\sqrt{3}}\right)^n$

What to know in Section 11.2

Exam 3 will cover only part of Section 11.2. You should be able to do problems like Problems 1-14 on pages 769-770. To prepare, start by reading the box in the middle of page 764 of the textbook and Examples 1 and 2 on page 764. Then read the section on “combining series” on pages 767-8.

Then do the HW problems: Problems 1-9 odd, 12 and 14 on pages 769-770. These problems are all done the same way:

(i) write out the series as a sum

(ii) identify A = initial term and r = ratio between successive terms

(iii) use the formula: $\text{sum} = \frac{A}{1-r}$.

Problems 12 and 14 require the idea of breaking the sum into two parts, each of which is a geometric series, as in Example 9 in the book.

Assorted Problems: On the exam, you will probably not be told which technique to apply. You should therefore practice doing some of the “assorted problems” — problems 151-220 on pages 637-9 of the textbook.

To do these problems, you must first identify the method (simple substitution, integration by parts, trig substitution, partial fractions). *Recommendation:* See if you can spot those problems that are solved by a trig substitution. Start to solve these, but stop after it is clear that your method will work.