

HW for Wednesday (these are copies of Problems 12, 3, 5, 7, 21, 24, 31, 32, 34, 37. in Section 10.5 of the textbook):
 Use the Ratio Test to determine if each series converges or diverges:

$$1. \sum_{n=1}^{\infty} \frac{2^n}{n!} \quad 2. \sum_{n=1}^{\infty} \frac{n+2}{3^n} \quad 3. \sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)^2} \quad 4. \sum_{n=1}^{\infty} \frac{n^4}{4^n} \quad 5. \sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n!3^{2n}}$$

Use any method to determine if each series converges or diverges Give reasons for your answer.

$$21. \sum_{n=1}^{\infty} \frac{n^{10}}{10^n} \quad 24. \sum_{n=1}^{\infty} \frac{(-2)^n}{3^n} \quad 31. \sum_{n=1}^{\infty} \frac{\ln n}{n} \quad 32. \sum_{n=1}^{\infty} \frac{n \ln n}{2^n} \quad 34. \sum_{n=1}^{\infty} n^3 e^{-n} \quad 37. \sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$$

Practice Problems for Math 133 Exam 3

1. Work out the following integrals using partial fractions.

$$(a) \int \frac{4x^2 + 6 dx}{x^3 + 3x} = 2 \ln x + \ln(x^2 + 3) + C$$

$$(b) \int \frac{x^2 + x - 10 dx}{(2x - 3)(x^2 + 4)} = \frac{1}{2} \ln \left(\frac{x^2 + 4}{2x - 3} \right) + \arctan \frac{x}{2} + C$$

$$(c) \int \frac{dx}{(x^3 + x^2 + x)} = -\frac{1}{2} \ln \left(\frac{x^2 + x + 1}{x^2} \right) - \frac{\sqrt{3}}{3} \arctan \left(\frac{2x + 1}{\sqrt{3}} \right) + C$$

$$(d) \int_0^1 \frac{5x dx}{(x+2)(x^2+1)} = \ln \frac{8}{9} + \frac{\pi}{4} = 0.667\dots$$

2. Work out the following improper integrals. Show all limits and justify your responses using the Direct Comparison Test (DCT) and Limit Comparison Test (LCT).

$$(a) \int_0^2 \frac{dx}{\sqrt{4-x^2}} = \frac{\pi}{2}$$

$$(f) \int_0^{\infty} \frac{x dx}{(x^2 + 4)^{3/2}} = \frac{1}{6}$$

$$(b) \int_0^{12} \frac{2x dx}{(x^2 - 16)^{2/3}} = 9 \cdot 16^{2/3}$$

$$(g) \int_1^{\infty} \frac{dx}{\sqrt{x}(\sqrt{x} + 1)} \text{ diverges.}$$

$$(c) \int_2^{\infty} \frac{dx}{\sqrt[3]{x^4 - 1}} \text{ converges.}$$

$$(h) \int_{-1}^1 -x \ln |x| dx \text{ diverges.}$$

$$(d) \int_{10}^{\infty} \ln(\ln x) \text{ diverges. (compare to } g(x) = 1)$$

$$(i) \int_0^5 \frac{1 + \sqrt{x}}{(x-2)} dx \text{ diverges.}$$

$$(e) \int_2^{\infty} \frac{dx}{x \ln x} \text{ diverges.}$$

$$(j) \int_2^4 \frac{\cos x dx}{x\sqrt{x^2 - 4}} \text{ converges.}$$

3. Determine if the following sequences converge or diverge. If the sequence converges find the limit. Justify your responses.

$$(a) a_n = \frac{n + (-1)^n}{n} \quad (b) a_n = \sqrt{\frac{n}{3n+1}} \quad (c) a_n = \frac{1}{(0.99)^n} \quad (d) a_n = \frac{3^n}{n^3}$$

$$(e) a_n = \sqrt[n]{n^5} \quad (f) a_n = (n+3)^{1/(n+4)} \quad (g) a_n = \frac{\ln n}{n^c}, \quad c > 0$$

4. Determine if the following sequences converge or diverge. If the sequence converges find the limit. Again, justify your responses.

$$(a) a_n = \left(1 + \frac{1}{n^2}\right)^n \quad (c) a_n = \frac{(\ln n)^4}{\sqrt{n}}$$

$$(b) a_n = \ln n - \ln(n+1) \quad (d) a_n = \left(\frac{4n+5}{4n-1}\right)^n$$

Hint: Think about the limit of e^{a_n}

5. Determine whether the following geometric series converge or diverge. If the series converges, find the limit. Again, justify your responses. For the repeating decimals, first write as a geometric series.

$$(a) .13\overline{4} \quad (e) \sum_{n=1}^{\infty} \frac{e^n}{\pi^n}$$

$$(b) 4.52\overline{159} \quad (f) \sum_{k=1}^{\infty} \frac{(-2)^k}{5^k}$$

$$(c) \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots \quad (g) \sum_{n=1}^{\infty} \frac{6}{5^n} - \left(\frac{1}{\sqrt{3}}\right)^n$$

$$(d) \sum_{n=1}^{\infty} \frac{\pi^n}{e^n}$$

6. Use the Integral Test (IT), Direct Comparison Test (DCT), Limit Comparison Test (LCT) or Ratio Test (RT) to determine whether the following series converge absolutely, converge conditionally, or diverge. Show your reasoning clearly and state the test you use.

$$(a) S = \sum_{n=1}^{\infty} \frac{1}{1+n^2} \quad (e) S = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n+5}}$$

$$(b) S = \sum_{n=1}^{\infty} \frac{3^n n!}{2n!} \quad (f) S = \sum_{n=1}^{\infty} \frac{\ln n}{n^2}$$

$$(c) S = \sum_{n=1}^{\infty} \frac{n^2 + 5n}{3n^3 - 14n^2 + 3} \quad (g) S = \sum_{n=1}^{\infty} \frac{n^6}{2^n}$$

$$(d) S = \sum_{n=1}^{\infty} \frac{\pi^n}{n(n+1)} \quad (h) S = \sum_{n=1}^{\infty} \frac{\sin n}{n^{3/2}}$$

Additional Review Problems: Page 432: 85, 89, 95, 96, 106. Page 491: 53, 57, 63, 65, 67, 71, 83, 85, 89. Pages 605-6: 1-13 odd, 23-26 all, 28, 30-32 all, 35, 38-40 all.