

Practice Problems for Math 133 Exam 4

Exam 4 will cover the material done in class and in homework from Sections 11.2 – 11.10 plus Sections 10.4 and 10.5. The problems on this sheet should help to remind you of this material. You should also go through your class notes and your HW problems. It is also especially helpful to go through your quizzes and check your answers against the solutions posted on the class web page.

1. Find the sum of the series $S = \sum_{n=1}^{\infty} \frac{\pi^n}{4^n}$ and the series $S = \sum_{n=1}^{\infty} \frac{3}{7^n} - \left(\frac{1}{\sqrt{2}}\right)^n$.

2. Use the Integral Test (IT), Limit Comparison Test (LCT), Comparison Test (CT), Ratio Test (RT) or the Alternating Series Test (AST) to determine whether the following series converge absolutely, converge conditionally, or diverge. Show your reasoning clearly and state the test you use.

(a) $S = \sum_{n=1}^{\infty} \frac{1}{1+n^2}$

(e) $S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

(b) $S = \sum_{n=1}^{\infty} \frac{3^n n!}{2n!}$

(f) $S = \sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

(c) $S = \sum_{n=1}^{\infty} \frac{n^2 + 5n}{3n^3 - 14n^2 + 3}$

(g) $S = \sum_{n=1}^{\infty} \frac{(-1)^n n^6}{2^n}$

(d) $S = \sum_{n=1}^{\infty} \frac{\pi^n}{n(n+1)}$

(h) $S = \sum_{n=1}^{\infty} \frac{\sin n}{n^{3/2}}$

3. Find the interval of convergence for the following power series (do not test for convergence at the endpoints).

(a) $\sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{2n+1}$ *Ans.* $1 < x < 3$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n+1} (x-5)^n$ *Ans.* $4 < x < 6$

(b) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{n!}$ *Ans.* all x

(d) $\frac{x}{1 \cdot 2} - \frac{x^2}{2 \cdot 2^2} + \frac{x^3}{3 \cdot 2^3} - \frac{x^4}{4 \cdot 2^4} + \dots$ *Ans.* $|x| < 2$

4. Use the general formula for Taylor Series to verify the following Maclaurin and Taylor series.

(a) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

(c) $\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \dots$

(b) $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$ (no pattern!)

(d) $\tan\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8x^3}{3} + \dots$

5. Use the degree 8 Maclaurin Polynomial $P_8(x)$ for the function $\sin x$ and a calculator to verify that the sin of 1 radian is

$$\sin 1 = 0.84146 \pm 0.000003$$

Hint: To get the error ± 0.000003 , use the formula for the remainder R_8 given on the latest *Series Tests* handout.

6. Use the six “Standard Taylor Series” (for e^x , $\sin x$, etc) that are listed on the *Series Tests* handout to figure out the Maclaurin Series for the following functions. *Hint:* You can substitute (e.g. replacing x by x^2) and you can differentiate and integrate term-by-term.

(a) $\frac{1}{1+x}$

(e) $\int_0^x \frac{\sin t}{t} dt$

(b) $\sin(x^2)$

(f) $\int_0^x e^{-t^2} dt$

(c) $5 \cos \pi x$

(d) $\cosh x = \frac{e^x + e^{-x}}{2}$

(g) $(1+x^2)^{1/2}$ (first 4 terms only).

7. Do the following problems from page 840 in the textbook (Chapter 11 review)

23-41 odd, 42, 43, 47, 57, 58, 59, 63, 65.

8. Finish the Supplementary Problems for Section 10.5 (download from class web page), doing the ones you did not do for HW.

Hints: Sketch the graph. If each x value occurs for at most 1 point on the graph, take $t = x$ and solve for y in terms of t . If each y value occurs for at most 1 point on the graph, take $t = y$ and solve for x in terms of t . For circles and ellipses, take t to be the central angle, so $x(t) = a \cos t$ and $y(t) = b \sin t$ for some numbers a, b .

9. Verify the following conversions between polar equations and Cartesian equations or vice versa. *Hints:* convert all trig functions into $\sin \theta$ and $\cos \theta$ and multiply by 1 in the form $\frac{r}{r}$ to create the expressions $x = r \sin \theta$ and $y = r \cos \theta$. Remember the identity for $\sin 2\theta$ and the property $\ln ab = \ln a + \ln b$.

(a) $r^2 \sin 2\theta = 2$

(e) $r^2 + 2r^2 \sin \theta \cos \theta = 1$

(b) $r = 4 \tan \theta \sec \theta$

(f) $xy = 5$

(c) $r = \csc \theta e^{r \cos \theta}$

(d) $r \sin \theta = \ln r + \ln \cos \theta$

(g) $x^2 - y^2 = 1$