

Practice Problems for Math 133 Exam 4

Exam 4 will cover the material done in class and in homework for Sections 10.6 – 10.10 and 11.1 – 11.5. You should also go through your class notes, your HW problems, and your quizzes. In particular, compare the style and method of your quiz solutions with the solutions posted on the class webpage. You should also do the Review Problems listed on the “Final Week” handout (some are repeated in the last problem below).

On Exam 4 you will be expected to use the convergence tests (Divergence Test, Integral Test, DCT, LCT and Ratio Test) learned earlier – See the “Complete Series” Handout.

1. Find the sum of the series $S = \sum_{n=1}^{\infty} \frac{\pi^n}{4^n}$ and the series $S = \sum_{n=1}^{\infty} \frac{3}{7^n} - \left(\frac{1}{\sqrt{2}}\right)^n$.

2. Use the Integral Test (IT), Limit Comparison Test (LCT), Comparison Test (CT), Ratio Test (RT) or the Alternating Series Test (AST) to determine whether the following series are absolutely convergent (AC), conditionally convergent (CC), or divergent (Div). Show your reasoning clearly and state the test you use.

(a) $S = \sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

(e) $S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\ln n}{n}$

(b) $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{n+5}$

(f) $S = \sum_{n=1}^{\infty} \frac{(-1)^n n^6}{2^n}$

(c) $S = \sum_{n=1}^{\infty} \frac{(-1)^n 3n}{8^n}$

(g) $S = \sum_{n=1}^{\infty} \frac{n^n}{n!}$

(d) $S = \sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$

3. Find the interval of convergence for the following power series (do not test for convergence at the endpoints).

(a) $\sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{2n+1}$ *Ans.* $1 < x < 3$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2n+1} (x-5)^n$ *Ans.* $4 < x < 6$

(b) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{n!}$ *Ans.* all x

(d) $\frac{x}{1 \cdot 2} - \frac{x^2}{2 \cdot 2^2} + \frac{x^3}{3 \cdot 2^3} - \frac{x^4}{4 \cdot 2^4} + \dots$ *Ans.* $|x| < 2$

4. Use the general formula for Taylor Series to verify the following Maclaurin and Taylor series.

$$(a) \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$(c) \sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \dots$$

$$(b) \tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots \text{ (no pattern!)} \quad (d) \tan\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8x^3}{3} + \dots$$

5. Use the degree 8 Maclaurin Polynomial $P_8(x)$ for the function $\sin x$ and a calculator to verify that the sin of 1 radian is

$$\sin 1 = 0.84146 \pm 0.000003$$

Hint: To get the error ± 0.000003 , use the formula for the remainder R_8 given on the latest *Series Tests* handout.

6. Use the six “Standard Taylor Series” (for e^x , $\sin x$, etc) that are listed on the *Series Tests* handout to figure out the Maclaurin Series for the following functions. *Hint:* You can substitute (e.g. replacing x by x^2) and you can differentiate and integrate term-by-term.

$$(a) \frac{1}{1+x}$$

$$(e) \int_0^x \frac{\sin t}{t} dt$$

$$(b) \sin(x^2)$$

$$(f) \int_0^x e^{-t^2} dt$$

$$(c) 5 \cos \pi x$$

$$(d) \cosh x = \frac{e^x + e^{-x}}{2}$$

$$(g) (1+x^2)^{1/2} \text{ (first 4 terms only).}$$

7. Convert from Cartesian to polar coordinates: (a) $xy = 5$ (b) $x^2 - y^2 = 1$.

8. Convert from polar to Cartesian coordinates:

$$(a) r^2 \sin 2\theta = 2$$

$$(c) r = \csc \theta e^{r \cos \theta}$$

$$(b) r = 4 \tan \theta \sec \theta$$

$$(d) r^2 + 2r^2 \sin \theta \cos \theta = 1$$

9. Do the following problems from Pages 655-7 in the textbook. (These are the Chapter 11 review problems listed as Monday’s HW on the “Final Week” handout.)

1, 3, 17, 18, 37, 39-46 all, 47, 49, 51, 53.