Solution to Problem 27 in Section 8.2: Find $\int_0^{\pi/3} x \tan^2 x \, dx$

$$\begin{aligned} \int x \tan^2 x \, dx &= \int x(\sec^2 x - 1) \, dx \\ &= \int x \sec^2 x \, dx - \int x \, dx \end{aligned}$$

Integrate by parts using $\boxed{\begin{array}{ll} u = x & dv = \sec^2 x \, dx \\ du = dx & v = \tan x \end{array}}$:

$$\begin{aligned} \int x \tan^2 x \, dx &= \left[x \tan x - \int \tan x \, dx \right] - \frac{x^2}{2} \\ &= x \tan x - \ln |\sec x| - \frac{x^2}{2} \end{aligned}$$

(the integral of tan is one of the integrals on our chart). We can also write $-\ln|\sec x|$ as $\ln|\cos x|$. Putting in the limits of integration then gives

$$\begin{aligned} \int_0^{\pi/3} x \tan^2 x \, dx &= \left(x \tan x + \ln|\cos x| - \frac{x^2}{2} \right|_0^{\pi/3} \\ &= \left(\frac{\pi}{3} \tan \frac{\pi}{3} + \ln \left| \cos \frac{\pi}{3} \right| - \frac{\pi^2}{18} \right) - (0 + \ln|\cos 0| - 0) \\ &= \frac{\pi}{3} \cdot \sqrt{3} + \ln \frac{1}{2} - \frac{\pi^2}{18}. \end{aligned}$$