Supplemental Exercises for Section 11.4

Which of the following series converge and which diverge? Verify your answer.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n+7}}$

2. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{3n-1}}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{6n+1}{n^3+4}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n\sqrt[3]{n^2+2}}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}\sqrt{n^2+5}}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n(n+10)}$

7. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{\frac{3}{2}}}{n(n+5)}$

Selected Answers

1. Because $\displaystyle\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}\sqrt{n+7}}}{\frac{1}{n}} = 1$ and because $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, by the Limit Comparison Test, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n-1}}$ diverges.

3. Because $\displaystyle\lim_{n\to\infty} \frac{\frac{6n+1}{n^3+4}}{\frac{1}{n^2}} = 6$ and because $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, by the Limit Comparison Test $\displaystyle\sum_{n=1}^{\infty} \frac{6n+1}{n^3+4}$ converges.

6. By using L'Hôpital's rule $\displaystyle\lim_{n\to\infty} \frac{\frac{\ln n}{n(n+10)}}{\frac{1}{n^{\frac{3}{2}}}} = 0$. Thus because $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ converges, by the Limit Comparison Test $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n(n+10)}$ converges.