

MTH 133

MAKEUP FINAL EXAM

DECEMBER 14, 2004

NAME: _____ STUDENT NUMBER: _____

SECTION NUMBER: _____ RECITATION INSTRUCTOR: _____

SCORE: _____

#####

1. Compute the derivative of each of the following functions. You do not need to simplify your answer.

(a) (9 pts) $y = x^2 \tan^{-1}(2x^3)$

(b) (9 pts) $y = \{\ln(e^x + 1)\}^3$

(c) (9 pts) $y = (\cos x)^{\sin x}$

2. Evaluate the following integrals. Please show your work and circle your answer.

(a) (9 pts) $\int \frac{1}{x^2 + 2x} dx$

(b) (9 pts) $\int \frac{dx}{(r^2 - x^2)^{3/2}} ; r > 0 , -r < x < r$

(c) (9 pts) $\int x e^{-2x} dx$

(d) (9 pts) $\int_1^{\infty} x^{-\frac{4}{3}} dx$

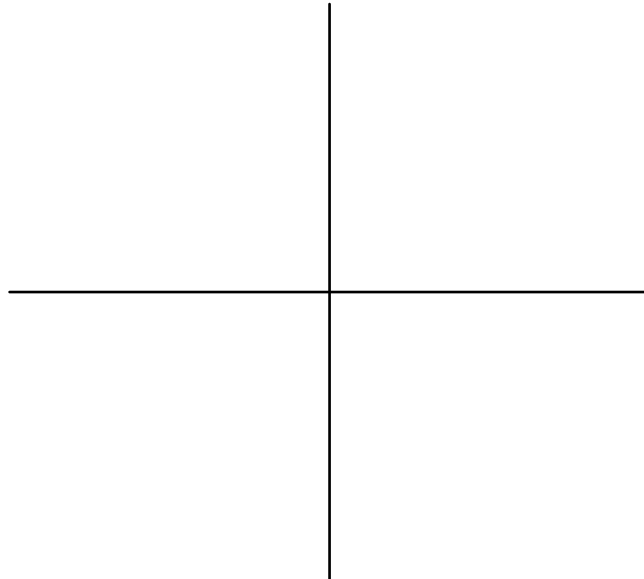
3. Evaluate the following limits. Please show your work.

(a) (8 pts) $\lim_{x \rightarrow 0^+} x^2 \ln x$

(b) (10 pts) $\lim_{x \rightarrow +\infty} \left(\frac{x}{x+1} \right)^x$

4. Consider the finite region in the first quadrant bounded by the curves $y = x^2$ and $y = 2x$.

(a) (5 pts) Sketch and shade the region.



Set up completely, but do not evaluate, an integral for the following quantities:

(b) (5 pts) The area of the region in (a).

(c) (5 pts) The volume of the solid obtained by rotating the region in (a) around the x-axis.

(d) (5 pts) The perimeter of the region in (a).

(e) (5 pts) The volume of the solid obtained by rotating the region in (a) around the line $x=2$.

5. Determine whether each of the series is convergent or divergent. Show your work and name the test(s) you are using.

(a) (8 pts) $\sum_{n=1}^{\infty} \frac{n^2 + 2n}{100n^2 + 5n - 3}$

(b) (8 pts) $\sum_{n=0}^{\infty} \frac{\sqrt{n} + 2}{3n^2 - n + 2}$

(c) (8 pts) $\sum_{k=1}^{\infty} \frac{3k^2}{2^k}$

(d) (8 pts) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

6. (12 pts) Determine the open interval of convergence of the series $\sum_{k=0}^{\infty} \frac{(x+2)^k}{(k+1)3^k}$.

7.(10 pts) Find the Maclaurin series expansion for $\frac{x}{1-x^2}$. Determine the open interval of convergence of the series.

8. (15 pts) A hemispherical tank with radius 10 feet (opening down, like a dome) is filled with water weighing 62.5 lbs/ft^3 . Find the amount of work required to pump all of the water out to a height of 6 feet above the top of the tank.

9. (10 pts) Solve the following differential equations.

$$\frac{dy}{dx} = y + xy \quad , \quad y(0) = 1$$

10. (15 pts) On the same polar coordinate system, sketch the graphs of the equations $r = 2 \sin \theta$ and $r = 1$, carefully locating all intersection points. Find the area inside the first curve and outside the second curve.