

Math 133 Final Exam, Spring 2006

Name: _____

Instructor: _____

PID: _____

Section: _____

Total: _____

Instructions: There are 10 pages, with a total of 200 possible points. You must show all necessary work to receive credit. Calculators are not allowed on this exam.

1. Find the derivative for the following functions.

(a) (10 points) $y = \sin^{-1}(x^2)$

(b) (10 points) $y = e^{\ln(4x^2+2)-\ln 2}$. Simplify your answer.

(c) (10 points) $y = (\sqrt{x})^x$. Write $\frac{dy}{dx}$ in terms of x only. You do not need to simplify.

2. (9 points) Find the solution of the differential equation $\frac{dy}{dx} = y \sinh x$ satisfying the condition $y(0) = 1$. Solve for y in explicit form. (That is, write y as a function of x .)

3. **Set up, but do not evaluate**, the integral for the volume of the solid generated by revolving the finite region bounded by $y = \sqrt{x}$, the x-axis and the line $x = 4$

(a) (6 points) about the x-axis (Include a rough graph of the rotated solid)

(b) (6 points) about the line $y = 3$ (Include a rough graph of the rotated solid)

4. (7 points) **Set up, but do not evaluate**, an integral for the length of the curve $y = e^{2x}$, $0 \leq x \leq 2$.

5. Solve each of the following. **Write your answer in terms of the original variable.**

(a) (8 points) $\int (\sec^2 x) e^{\tan x} dx$

(b) (10 points) $\int \frac{2}{x(x^2 + 2)} dx$

(c) (12 points) $\int \frac{x^3}{\sqrt{1-x^2}} dx$

6. (10 points) Evaluate $\int_0^1 \frac{\tan^{-1} x}{x^2 + 1} dx$

7. (10 points) Tell whether the improper integral $\int_0^{\infty} xe^{-x} dx$ converges or diverges, and if possible, give its value.

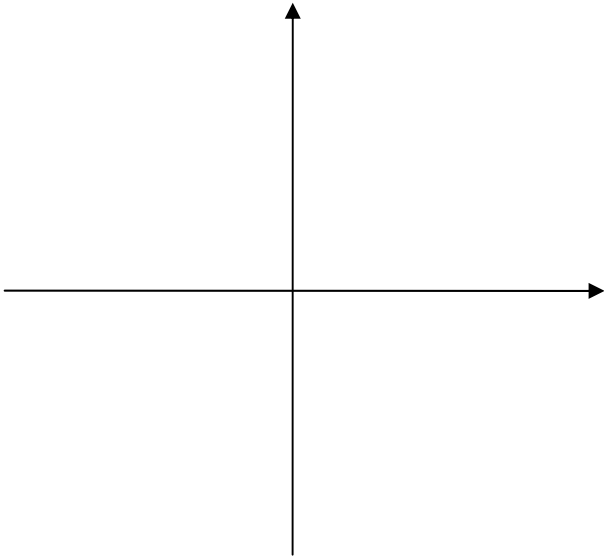
8. (10 points) An unstable isotope of plutonium has a half life of 80 years. How long will it take for 100 grams of this isotope to decay to 30 grams?

9. (9 points) Determine the open interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x+2)^n}{2^n(n+1)}$. You do not need to check the endpoints.

10. (9 points) Write down the first three terms of the (convergent) series $\sum_{n=1}^{\infty} (-1)^n \frac{5}{4^n}$, and then calculate the sum of the series.

11. Consider the curves given in polar coordinates by $r = 2 \sin 2\theta$ and $r = 1$, where $0 \leq \theta \leq \frac{\pi}{2}$.

(a) (8 points) Sketch both curves on the same coordinate system, locating the points of intersection.



(b) (8 points) Set up, but do not evaluate, an integral for the area of the region in the first quadrant inside the curve $r = 2 \sin 2\theta$ and outside the curve $r = 1$.

12. (8 points each) Verify that the series converges, or explain why it diverges.

(a)
$$\sum_{n=1}^{\infty} \frac{2n}{n^2 + 1}$$

(b)
$$\sum_{n=1}^{\infty} \cos\left(\frac{1}{n^2}\right)$$

(c)
$$\sum_{n=1}^{\infty} \frac{n^2 2^n}{n!}$$

(d)
$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

13. (6 points) Find the Maclaurin series for the function $f(x) = \frac{1}{3+2x}$. Express your answer in summation notation.

14. (10 points) Find the Taylor polynomial $P_2(x)$ and Taylor's form of the remainder $R_2(x)$ for $f(x) = \sqrt{x}$ (Take $a = 1$).