Math 133 — Introduction to series worksheet

Directions: *Read, filling in the blanks as you go.*

A *series* is an sum of infinitely many numbers. For example,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots \tag{1}$$

where the dots indicate that we continue, summing infinitely many terms according to the pattern indicated (powers of 2 in the denominator). The total sum is calculated by adding up the first 10 terms, then the first 100, then the first 1000 (these are called the "partial sums") and then taking a limit (this is made more precise below).

Problem 1 Compute the partial sums of the series (1) above by filling in the blanks. *Write your answers as fractions.*

$s_1 =$ first term $=$ _____ $\qquad$ $s_2 =$ sum of the first two terms $=$ _____

In general, we write $s_n =$ sum of the first n terms (the s stands for "sum"). Keep going, still writing as fractions:

$s_3 =$ _____	$s_6 =$ _____
$s_4 =$ _____	$s_7 =$ _____
$s_5 =$ _____	$s_8 =$ _____

These partial sums seem to be approaching the number _____. In fact, we can write the n^{th} partial sum as a fraction with denominator 2^n, namely

$$s_n = \_\_\_\_\_$$

and then take the limit:

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \qquad\qquad =$$

(fill in to take the limit; it helps to rewrite by dividing numerator and denominator by 2^n).

Definition If the partial sums converge to a limit S we say the series *converges* and that its *sum* is S. If the parial sums do not converge we say the series *diverges*.

Problem 2 Consider the series

$$\frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625} + \cdots \tag{2}$$

(the denominators are powers of 5). Write the partial sums in the form $\frac{1}{2} -$ (something) and, based on the pattern, guess a formula for s_n:

$$s_1 = \frac{2}{5} = \frac{1}{2} - \frac{}{10} \qquad s_4 = \underline{\qquad} = \frac{1}{2} - \underline{\qquad}$$

$$s_2 = \underline{\qquad} = \frac{1}{2} - \frac{}{50} \qquad s_n = \underline{\qquad\qquad}$$

$$s_3 = \underline{\qquad} = \frac{1}{2} - \underline{\qquad}$$

Since $\lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1}{2} - \underline{\qquad} = \underline{\qquad}$, the series (2) converges and its sum is $S = \underline{\qquad}$.

Problem 3 For the series

$$\frac{1}{17} + \frac{1}{17} + \frac{1}{17} + \frac{1}{17} + \frac{1}{17} + \cdots \tag{3}$$

(all terms are the same) the n^{th} partial sum is $s_n = \underline{\qquad}$. Because

$$\lim_{n\to\infty} s_n = \lim_{n\to\infty} \underline{\qquad\qquad} = \underline{\qquad},$$

the series (3) ______(converges or diverges).

Problem 4 For each whole number n we can find another number $n!$ (pronounced "n factorial") by multiplying together all the whole numbers up to n:

$$n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots n$$

Using a calculator, find these factorials:

$2! = 1 \cdot 2 =$ ______ $\qquad 5! =$ ______ $\qquad 8! =$ ______

$3! = 1 \cdot 2 \cdot 3 =$ ______ $\qquad 6! =$ ______ $\qquad 9! =$ ______

$4! = 1 \cdot 2 \cdot 3 \cdot 4 =$ ______ $\qquad 7! =$ ______ $\qquad 10! =$ ______

Problem 5 Now consider the series

$$1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \cdots \tag{4}$$

Using a calculator, write down the partial sums as decimals (record 6 digits after the decimal point):

s_3 = __________	s_6 = __________	s_9 = __________
s_4 = __________	s_7 = __________	s_{10} = __________
s_5 = __________	s_8 = __________	s_{11} = __________

The series (4) seems to converge and the sum appears to be the number S = _____ (you should recognize the number s_{11}).

Problem 6 Now modify the series in the previous problem by making the signs alternate:

$$1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!} - \cdots \tag{5}$$

Again, using a calculator, write down the partial sums as decimals (record 6 digits after the decimal point):

s_3 = __________	s_6 = __________	s_9 = __________
s_4 = __________	s_7 = __________	s_{10} = __________
s_5 = __________	s_8 = __________	s_{11} = __________

Compare these partial sums to the numbers (write with 6 decimal places):

e = __________ and $\frac{1}{e}$ = __________

The series (5) seems to converge and the sum appears to be S = _____.

Problem 7 Now consider the series

$$\frac{2}{1 \cdot 3^1} + \frac{2}{3 \cdot 3^3} + \frac{2}{5 \cdot 3^5} + \frac{2}{7 \cdot 3^7} + \frac{2}{9 \cdot 3^9} + \frac{2}{11 \cdot 3^{11}} + \cdots \tag{6}$$

(the numerator is always 2 and the denominators are odd numbers times the same odd power of 3). This series converges. Find the sum of the terms shown and compare to $\ln 2$ = __________ (write to 6 decimal places).

As before, write the partial sums as decimals to 6 places. *Stop when you reach a partial sum that is equal to* $\ln 2$ *to 6 decimal places.*

s_1 = __________	s_4 = __________	s_7 = __________
s_2 = __________	s_5 = __________	s_8 = __________
s_3 = __________	s_6 = __________	s_9 = __________