

Math 133 – Polar Zoo

Four types of figures commonly occur in calculus problems involving polar coordinates.

Circles

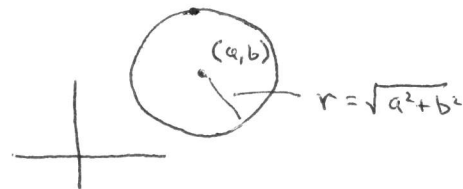


$$r = 1$$



$$r = 2 \sin \theta$$

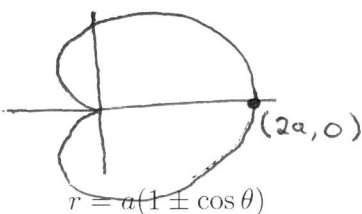
$$r = 2 \cos \theta$$



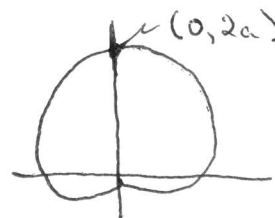
$$r = 2a \cos \theta + 2b \sin \theta$$

In the third case, multiply both sides by r to get $x^2 + y^2 = 2ax + 2by$, then complete the square to get $(x - a)^2 + (y - b)^2 = a^2 + b^2$, which is a circle with center (a, b) and radius $\sqrt{a^2 + b^2}$.

Cardioids

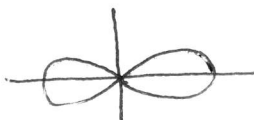


$$r = a(1 \pm \cos \theta)$$



$$r = a(1 \pm \sin \theta)$$

Lemniscates



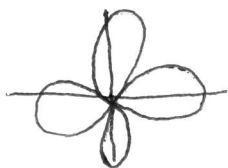
$$r^2 = \cos 2\theta$$



$$r^2 = \sin 2\theta$$

Roses

Even-leafed:



$$r = \cos 2\theta$$



$$r = \sin 2\theta$$



$$r = \sin 4\theta$$

Odd-leafed:

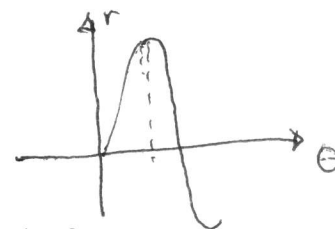


$$r = \cos 3\theta$$



$$r = \sin 5\theta$$

To draw roses, sketch graph in $\theta - r$ plane and identify the first maximum. (Similarly for lemniscates.)



Ex

$$r = \sin 3\theta$$

1st maximum at $\frac{90^\circ}{3} = 30^\circ$