SOLUTIONS — See "Advise" on next page

MATH 133 – Practice Multiple Choice Questions for Final Exam

- For each multiple choice problem below, write the letter of your answer in the blank provided; partial credit is possible for some answers.
- Show your work on all problems, including multiple choice problems.

1. Evaluate: $\int_1^2 \frac{dx}{x(x^2+1)}$. Write the letter of the best answer here: H

(A) $\ln\left(\frac{2}{\sqrt{5}}\right)$ (B) $\ln\left(\frac{1}{\sqrt{2}}\right)$ (C) $\ln\left(\frac{3}{\sqrt{2}}\right) - \ln\left(\frac{5}{\sqrt{2}}\right)$ (D) $\ln\left(\frac{7}{\sqrt{3}}\right) - \ln\left(\frac{1}{\sqrt{5}}\right)$ (E) $\ln\left(\frac{3}{\sqrt{2}}\right)$

(F) $\ln\left(\frac{7}{\sqrt{2}}\right)$ (G) $\ln\left(\frac{5}{\sqrt{5}}\right)$ (H) $\ln\left(\frac{2}{\sqrt{5}}\right) + \ln\left(\sqrt{2}\right)$ (I) $\ln\left(\frac{2}{\sqrt{7}}\right) + \ln\left(\frac{3}{\sqrt{2}}\right)$ (J) $\ln 3$

$$\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1) + (Bx+C)x}{x(x^2+1)}$$

$$0\cdot x^2 + 0\cdot x + 1 = A(x^2+1) + Bx^2 + Cx = (A+B)x^2 + Cx + A$$

Match coefficients: $A=1$, $C=0$, $B=-1$.

$$\int_1^2 \frac{dx}{x(x^2+1)} = \int_1^2 \frac{dx}{x} - \int_1^2 \frac{x}{x^2+1}\,dx = \ln x\Big|_1^2 - \frac{1}{2}\int \frac{du}{u} = \ln x - \frac{1}{2}\ln(x^2+1)\Big|_1^2$$

$$\left[\begin{array}{c} u = x^2+1 \\ du = 2x\,dx \\ \frac{1}{2}du = x\,dx \end{array}\right]$$

$$= \ln 2 - \ln 1^{\to 0} - \frac{1}{2}\ln 5 + \frac{1}{2}\ln 2$$

$$= \frac{3}{2}\ln 2 - \ln\sqrt{5}$$

$$= \ln 2\sqrt{2} - \ln\sqrt{5} = \text{(H)}$$

2. Determine whether the integral $I = \int_0^1 x \ln x\,dx$ is convergent (**conv**) or divergent (**div**). If it is convergent, evaluate it. Write the letter of the best answer here: G

(A) conv, $I=\frac{1}{8}$ (B) conv, $I=\frac{1}{2}$ (C) conv, $I=1$ (D) conv, $I=-\frac{1}{2}$ (E) conv, $I=-1$

(F) conv, $I=2$ (G) conv, $I=-\frac{1}{4}$ (H) conv, $I=4$ (I) conv, $I=-4$ (J) div

$$\left[\begin{array}{ll} u = \ln x & dv = x \\ du = \frac{1}{x}dx & v = \frac{1}{2}x^2 \end{array}\right]$$

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \int \frac{1}{2}x\,dx = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$$

$$\Rightarrow I = \lim_{a\to 0}\int_a^1 x\ln x\,dx = \lim_{a\to 0}\frac{x^2}{2}\ln x\Big|_a^1 - \frac{1}{4}\lim_{a\to 0} x^2\Big|_a^1$$

$$= \frac{1}{2}\ln 1^{\to 0} - \left[\frac{1}{2}\lim_{a\to 0} a^2\ln a\right] - \frac{1}{4}\left[1 - \underbrace{\lim_{a\to 0} a^2}_{=0}\right]$$

$$= \frac{1}{2}\lim_{x\to 0}\frac{\ln x}{1/x^2} \quad \text{type } \frac{\infty}{\infty}$$

$$\overset{\text{L'Hop}}{=} \frac{1}{2}\lim_{x\to 0}\frac{1}{x}\cdot\frac{1}{-2/x^3} = \frac{1}{2}\lim_{x\to 0}\frac{x^2}{-2} = 0.$$

$$= -1/4. = \text{(G)}$$

3. Solve the differential equation $\frac{dy}{dx} = xy^2$ with initial condition $y(0) = -1$.

Write the letter of the best answer here: I

(A) $\frac{2}{x^3-2}$ (B) $\frac{5}{x^2-5}$ (C) $\frac{-2}{x+2}$ (D) $\frac{2}{x^4-2}$ (E) $\frac{4}{x^4-4}$

(F) $\frac{-4}{x+4}$ (G) $\frac{3}{x^3-3}$ (H) $\frac{1}{x^4-1}$ (I) $\frac{-2}{x^2+2}$ (J) $\frac{-5}{x+5}$

Separate variables: $y^{-2}\,dy = x\,dx$.

Integrate $\int y^{-2}\,dy = \int x\,dx$

$-y^{-1} = \frac{x^2}{2} + C$

Put in $x=0$, $y=-1$ $\quad 1 = 0 + C \quad$ (C=1)

Solve for y: $\quad -\frac{1}{y} = \frac{x^2}{2} + 1 = \frac{x^2+2}{2}$

$-y = \frac{2}{x^2+2}$

$y = \frac{-2}{x^2+2} =$ (I)

Advise

- Don't "eliminate" — do the problem as if it weren't multiple choice, then find your answer.
- Write your letter answer in the blank provided
- Show all work! Letter answers without work will be given zero points