Math 851: Numerical Analysis II

Homework Assignment Four

Spring 2009

1. Let G: $R^1 \to R^1$ be defined by $G(x) = x - x^3$. Show that $x^* = 0$ is a point of attraction of Φ: $x^{k+1} = G(x^k)$, $k = 0, 1, \cdots$, although $G'(x^*) = 1$. Show also that $R_1(\Phi, x^*) = Q_1(\Phi, x^*) = 1$. On the other hand, if $G(x) = x + x^3$, show that $x^* = 0$ is not a point of attraction.

2. Define G: $R^2 \to R^2$ by $g_1(x) = x_1^2 - x_2$, $g_2(x) = x_2^2$. Show that $x^* = 0$ is a point of attraction of Φ: $x^{k+1} = G(x^k)$ and that $R_1(\Phi, x^*) = 0$, but that $Q_1(\Phi, x^*) > 0$ in any norm.

3. Define G: $R^1 \to R^1$ by $G(x) = x^p$ for some $p > 1$. Show that $x^* = 0$ is a point of attraction of Φ: $x^{k+1} = G(x^k)$ and that $O_R(\Phi, x^*) = O_Q(\Phi, x^*) = p$.

4. Consider the Newton iteration applied to the mapping f: $R^1 \to R^1$ defined by $f(t) = \exp(-t^{-2})$, $t \neq 0$, $f(0) = 0$. Show that 0 is a point of attraction, but that $R_1(\Phi, 0) = Q_1(\Phi, 0) = 1$.

5. Define f: $R^1 \to R^1$ by $f(x) = x + x^{1+\alpha}$ for some $\alpha \in (0, 1]$. Show that $O_R(\Phi, 0) = O_Q(\Phi, 0) = 1 + \alpha$ for the Newton iteration.

Due date: Friday, Feb. 20, 2009. In class